CHAPTER 1: INTRODUCTION

Short Answer Problems

1.1 True: The earth is taken to be non-accelerating for purposes of modeling systems on the surface of the earth.

1.2 False: Systems undergoing mechanical vibrations are not subject to nuclear reactions is an example of an implicit assumption.

1.3 True: Basic laws of nature can only be observed and postulated.

1.4 False: The point of application of surface forces is on the surface of the body.

1.5 False: The number of degree of freedom necessary to model a mechanical system is unique.

1.6 False: Distributed parameter systems are another name for continuous systems.

1.7 True: The Buckingham Pi theorem states that the number of dimensionless variables required in the formulation of a dimensional relationship is the number of dimensional variables, including the dependent variable, minus the number of dimensions involved in the dimensional variables.

1.8 True: The displacement of its mass center (x and y coordinates) and the rotation about an axis perpendicular to the mass center are degrees of freedom the motion of an unconstrained rigid body undergoing planar motion.

1.9 False: A particle traveling in a circular path has a velocity which is tangent to the circle.

1.10 False: The principle of work and energy is derived from Newton's second law by integrating the dot product of the law with a differential displacement vector as the particle moves from one location to another.

1.11 The continuum assumption treats all matter as a continuous material and implies that properties are continuous functions of the coordinates used in modeling the system.

1.12 An explicit assumption must be stated every time it is used, whereas an implicit assumption is taken for granted.

1.13 Constitutive equations are used to model the stress-strain relationships in materials. They are used in vibrations to model the force-displacement relationships in materials that behave as a spring.

1.14 A FBD is a diagram of a body abstracted from its surroundings and showing the effects of the surroundings as forces. They are drawn at an arbitrary time.

1.15 The equation represents simple harmonic motion

1.16 (a) X is the amplitude of motion; (b) ω is the frequency at which the motion occurs (c) ϕ is the phase between the motion and a pure sinusoid.

1.17 The phase angle is positive for simply harmonic motion. Thus the response lags a pure sinusoid.

1.18 A particle has mass that is concentrated at a point. A rigid body has a distribution of mass about the mass center.

1.19 A rigid body undergoes planar motion if (1) the path of its mass center lies in a plane and (2) rotation occurs only about an axis perpendicular to the plane of motion of the mass center.

1.20 The acceleration of a particle traveling in a circular path has a tangential component that is the radius of the circle times the angular acceleration of the particle and a centripetal acceleration which is directed toward the center of the circle which is the radius time the square of the angular velocity.

1.21 An observer fixed at A observes, instantaneously that particle B is moving in a circular path of radius $|\mathbf{r}_{B/A}|$ about A.

1.22 It is applied to the FBD of the particle.

1.23 The effective forces for a rigid body undergoing planar motion are a force applied at the mass center equal to $m\bar{\mathbf{a}}$ and a moment equal to $\bar{I}\alpha$.

1.24 The two terms of the kinetic energy of a rigid body undergoing planar motion are $\frac{1}{2}m\bar{v}^2$, the translational kinetic energy, and $\frac{1}{2}\bar{I}\omega^2$, the rotational kinetic energy.

1.25 The principle of impulse and momentum states that a body's momentum (linear or angular) momentum at t_1 plus the external impulses applied to the body (linear or angular) between t_1 and t_2 is equal to the system's momentum (linear or angular) at t_2 .

1.26 One, let θ be the angular rotation of the bar, measured positive counterclockwise, from the system's equilibrium position.

1.27 Four, let x_1 be the absolute displacement of the cart, x_2 the displacement of the leftmost block relative to the cart, x_3 the displacement of the rightmost block away from the cart and θ the counterclockwise angular rotation of the bar.

1.28 Four, let x_1 represent the displacement of the center of the disk to the right, x_2 the downward displacement of the hanging mass, x_3 the displacement of the sliding mass to the left and θ the counterclockwise angular rotation of the rightmost pulley.

1.29 Two, let θ be clockwise the angular displacement of the bar and x the downward displacement of the hanging mass.

1.30 Three, let x be the downward displacement of the middle of the upper bar, θ its clockwise angular rotation and ϕ the clockwise angular rotation of the lower bar.

1.31 Three, let θ represent the clockwise angular rotation of the leftmost disk, ϕ the clockwise angular rotation of the rightmost disk and x the upward displacement of the leftmost hanging mass.

1.32 Infinite, let x be a coordinate measured along the neutral axis of the beam measured for the fixed support. Then the displacement is a continuous function of x and t, w(x,t).

1.33 Three, let x_1 be the downward displacement of the hand, x_2 the downward displacement of the palm and x_3 the displacement of the fingers.

1.34 Given: Uniform acceleration, a=2 m/s. (a) $v(t) = at + v_0 \Rightarrow v(5) = \left(2\frac{m}{s^2}\right)(5\text{ s}) + \left(0\frac{\text{m}}{\text{s}}\right) = 10\text{ m}$ (b) $x(t) = a\frac{t^2}{2} + v_0 t + x_0 \Rightarrow x(5) = \frac{1}{2}\left(2\frac{\text{m}}{\text{s}^2}\right)(5\text{ s})^2 = 25\text{ m}$

1.35 Given: $\mathbf{v} = 2\cos 2t\,\mathbf{i} + 3\sin 2t\,\mathbf{j} + 0.4\,\boldsymbol{k}$ m/s. (a) $\mathbf{a} = \frac{d\mathbf{v}}{dt} = -4\sin 2t\ \mathbf{i} + 6\cos 2t\ \mathbf{j}\ \text{m/s}^2 \Rightarrow a(\pi) = -4\sin 2\pi\ \mathbf{i} + 6\cos 2\pi\ \mathbf{j}\frac{\text{m}}{\text{s}^2} = 6\,\mathbf{j}\ \text{m/s}^2$ (b) $\mathbf{r} = \int \mathbf{v}dt = \left[(\sin 2t + C_1)\mathbf{i} + \left(-\frac{3}{2}\cos 2t + C_2\right)\mathbf{j} + (0.4t + C_3)\mathbf{k}\right]$ m. The particle starts at the origin at t = 0. Application of this condition leads to) $\mathbf{r}(\text{t}) = \left[(\sin 2t\,)\mathbf{i} + \left(-\frac{3}{2}\cos 2t + \frac{3}{2}\right)\mathbf{j} +\right.$ $0.4t\mathbf{k}$ m. Evaluation at π leads to $\mathbf{r}(\pi)=\sin 2\pi\,\mathbf{i}+-32\cos 2\pi+32\mathbf{j}+0.4\pi\mathbf{k}$ m $=0.4\pi\mathbf{k}$ m.

1.36 Given: v=2 m/s, r=3 m, $\theta(0) = 0$ (a) $v = \frac{ds}{dt} \Rightarrow s = \int v dt = 2t$ at t=2 s the particle has traveled 4 m. But $s = r\theta$ thus $\theta = \frac{4\text{ m}}{3\text{ m}} = 1.33\text{ rad} = 76.2°$. (b) The acceleration of a particle traveling on a circular path has two components. One is $\frac{dv}{dt}$ which is tangent to the circle and is zero for this problem. The other component is $\frac{v^2}{r} = \frac{(2\text{ m/s})^2}{3\text{ m}} = 1.33\text{ m}$ directed toward the center of the circle from the position of the particle.

1.37 Given: m=2 kg,$\bar{I} = 0.5\text{ kg}\cdot\text{m}^2$,$\bar{\mathbf{a}} = (5\mathbf{i} + 3\mathbf{j})\ \text{m/s}^2$, $\alpha = 10\text{ rad/s}^2$. Effective forces are $m\bar{\mathbf{a}} = (2\text{ kg})\left[(5\mathbf{i} + 3\mathbf{j})\frac{\text{m}}{\text{s}^2}\right] = (10\mathbf{i} + 15\mathbf{j})\frac{\text{m}}{\text{s}^2}$ applied at the mass center and a couple $\bar{I}\alpha = (0.5\text{ kg}\cdot\text{m}^2)(10\text{ rad/s}^2) = 5\text{ N}\cdot\text{m}$.

1.38 Given: m = 0.1 kg, $\mathbf{v} = (9\mathbf{i} + 11\text{j})\ \text{m/s}$. The kinetic energy of the particle is $T = \frac{1}{2}m|\mathbf{v}|^2 = \frac{1}{2}(0.1\text{ kg})\left(\sqrt{9^2 + 11^2}\ m/s\right)^2 = 0.711$ J.

1.39 Given: m=3 kg, $\bar{\mathbf{v}} = (3\mathbf{i} + 4\mathbf{j})$ m/s, d=0.2 m The angular velocity is calculated from $|\bar{\mathbf{v}}| = d\omega \Longrightarrow \omega = \frac{|\bar{\mathbf{v}}|}{d} = \frac{5\text{ m/s}}{0.2\text{ m}}$=20 rad/s.

1.40 Given: $= 100$ J , $I = 0.03\text{ kg}\cdot\text{m}^2$ The kinetic energy of a rigid body which rotates about its centroidal axis is $T = \frac{1}{2}I\omega^2$. Thus $100\text{ J} = \frac{1}{2}(0.03\text{ kg}\cdot\text{m}^2)\omega^2$ which leads to $\omega = 81.65\ \frac{\text{rad}}{\text{sec}}$.

1.41 Given: m = 5 kg, $\bar{v} = 4$ m/s, $\omega = 20$ rad/s, $\bar{I} = 0.08\text{ kg}\cdot\text{m}^2$. The kinetic energy of a rigid body undergoing planar motion is $T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}(5\text{ kg})(4\text{ m/s})^2 + \frac{1}{2}(0.08\text{ kg}\cdot\text{m}^2)(20\text{ rad/s})^2 = 56\text{ J}$.

1.42 Given: F=12,000 N, $\Delta t = 0.03$ s. The impulse applied to the system is $I = F\Delta t = (12{,}000\text{ N})(0.03\text{ s}) = 360\text{ N}\cdot\text{s}$.

1.43 Given: m = 3 kg, $v_1 = 0$ m/s, force as given in Figure (a) The impulse imparted to the particle is $I = \int_0^{3\text{ s}} Fdt = \frac{1}{2}(1)(100) + 2(100) + \frac{1}{2}(1)(100) = 300\text{ N}\cdot\text{s}$ (b) The velocity at t=2 s is given by the principle of impulse and momentum $mv = \int_0^{2\text{ s}} Fdt \Longrightarrow v = \frac{\int_0^{2\text{ s}} Fdt}{m} = \frac{250\text{ N}\cdot\text{s}}{3\text{ kg}} = 83.3\text{ m/s}$. (c) The velocity after 5 s is $v = \frac{\int_0^{5\text{ s}} Fdt}{m} = \frac{300\text{ N}\cdot\text{s}}{3\text{ kg}} = 100\text{ m/s}$.

1.44 Given: m = 2 kg, F=6 N, t=10 s, $v_1 = 4$ m/s. The principle of work and energy is used to calculate how far the particle travels $T_1 + U_{1\to 2} = T_2$ after the velocity $v_2$is calculated from the principle of impulse and momentum $mv_1 + I = mv_2 \Longrightarrow v_2 = \frac{mv_1 + I}{m} = \frac{(2\text{ kg})(4\text{ m/s}) + (6\text{ N})(10\text{ s})}{2\text{ kg}} = 34\text{ m/s}$. Then letting x be the distance traveled application of work and energy gives $\frac{1}{2}(2\text{ kg})(4\text{ m/s})^2 + (6\text{ N})x = \frac{1}{2}(2\text{ kg})(34\text{ m/}$ s2which is solved to yield x=190 m.

1.45 (a) -(ii) (b)-(iv) (c)-(i) (d)-(v) (e)-(i) (f)-(v) (g)-(vi) (h)-(iii) (i)-(ix)

Chapter Problems

1.1 The one-dimensional displacement of a particle is

$$x(t) = 0.5e^{-0.2t} \sin 5t \text{ m}$$

(a) What is the maximum displacement of the particle? (b) What is the maximum velocity of the particle? (c) What is the maximum acceleration of the particle?

Given: x(t)

Find: $(a) x_{max}$ $(b) v_{max}$ $(c) a_{max}$

Solution: (a) The maximum displacement occurs when the velocity is zero. Thus

$$\dot{x}(t) = 0.5e^{-0.2t}(-0.2 \sin 5t + 5 \cos 5t)$$

Setting the velocity to zero leads to

$$-0.2 \sin 5t + 5 \cos 5t = 0$$

or $\tan 5t = 25$. The first time that the solution is zero is t=0.3062. Substituting this value of t into the expression for x(t) leads to

$$x_{max} = 0.4699 \text{ m}$$

(b) The maximum velocity occurs when the acceleration is zero

$$\ddot{x}(t) = 0.5e^{-0.2t}[-0.2(-0.2 \sin 5t + 5 \cos 5t) - \cos 5t - 25 \sin 5t]$$
$$= 0.5e^{-0.2t}(-24.96 \sin 5t - 6 \cos 5t)$$

The acceleration is zero when $24.96 \sin 5t - 6 \cos 5t = 0 \Longrightarrow \tan 5t = -0.240$. The first time that this is zero is t=0.5812 which leads to a velocity of

$$v_{min} = -2.185 \text{ m/s}$$

(c) The maximum acceleration occurs when $\dddot{x} = 0$,

$$\dddot{x} = 0.5e^{-0.2t}[-0.2(-24.96 \sin 5t - 6 \cos 5t) - (24.96)(5) \cos 5t + 30 \sin 5t]$$
$$= 0.5e^{-0.2t}(34.992 \sin 5t - 123.6 \cos 5t)$$

The maximum acceleration occurs when $34.992 \sin 5t - 123.6 \cos 5t = 0 \Longrightarrow \tan 5t = 3.53$. The time at which the maximum acceleration occurs is t=0.2589 s which leads to

$$a_{max} = -12.18 \text{ m/s}^2$$

Problem 1.1 illustrates the relationships between displacement, velocity and acceleration.

1.2 The one-dimensional displacement of a particle is

$$x(t) = 0.5\,e^{-0.2t}\sin(5t + 0.24) \text{ m} \qquad (1)$$

(a) What is the maximum displacement of the particle? (b) What is the maximum velocity of the particle? (c) What is the maximum acceleration of the particle?

Given: x(t)

Find: $(a) x_{max}$ $(b) v_{max}$ $(c) a_{max}$

Solution: (a) The maximum displacement occurs when the velocity is zero. Thus

$$\dot{x}(t) = 0.5e^{-0.2t}[-0.2\sin(5t + 0.24) + 5\cos(5t + 0.24)]$$

Setting the velocity to zero leads to

$$-0.2\sin(5t + 0.24) + 5\cos(5t + 0.24) = 0$$

or $\tan(5t + 0.24) = 0.2582$. The first time that the solution is zero is t=0.3062. Substituting this value of t into the expression for x(t) leads to

$$x_{max} = 0.4745 \text{ m}$$

(b) The maximum velocity occurs when the acceleration is zero

$$\begin{aligned}\ddot{x}(t) = 0.5e^{-0.2t}\{&-0.2[(-0.2\sin(5t + 0.24) + 5\cos(5t + 0.24))] \\ &- \cos(5t + 0.24) - 25\sin(5t + 0.24)\} \\ = {}&0.5e^{-0.2t}[-24.96\sin(5t + 0.24) - 6\cos(5t + 0.24)]\end{aligned}$$

The acceleration is zero when

$$-24.96\sin(5t + 0.24) - 6\cos(5t + 0.24) = 0 \Longrightarrow \tan(5t + 0.24) = -0.240.$$

The first time that this is zero is t = 0.5332 which leads to a velocity of

$$v_{min} = -2.0188 \text{ m/s}$$

(c) The maximum acceleration occurs when $\dddot{x} = 0$,

$$\begin{aligned}\dddot{x} = 0.5e^{-0.2t}\{&-0.2[-24.96\sin(5t + 0.24) - 6\cos(5t + 0.24)] \\ &- (24.96)(5)\cos(5t + 0.24) + 30\sin(5t + 0.24)\} \\ = {}&0.5e^{-0.2t}[34.992\sin(5t + 0.24) - 123.6\cos(5t + 0.24)]\end{aligned}$$

The maximum acceleration occurs when

$$34.992\sin(5t + 0.24) - 123.6\cos(5t + 0.24) = 0 \Longrightarrow \tan(5t + 0.24) = 3.53.$$

The time at which the maximum acceleration occurs is t=0.2109 s which leads to

$$a_{max} = -12.30 \text{ m/s}^2$$

Problem 1.2 illustrates the relationships between displacement, velocity and acceleration.

1.3 At the instant shown in Figure P1.3, the slender rod has a clockwise angular velocity of 5 rad/sec and a counterclockwise angular acceleration of 14 rad/sec^2. At the instant shown, determine (a) the velocity of point *P* and (b) the acceleration of point *P*.

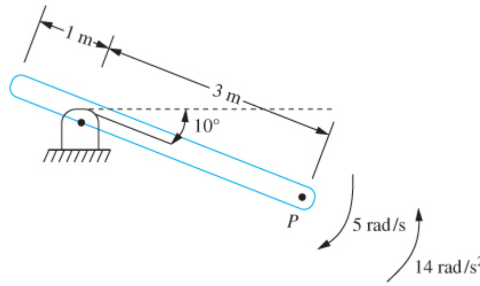


FIGURE P1.3

Given: $\omega = 5$ rad/sec, $\alpha = 14$ rad/sec^2, $\theta = 10°$

Find: v_p, a_P

Solution: The particle at the pin support, call it O, is fixed. Hence its velocity and acceleration are zero. Using the relative velocity and acceleration equations between two particles on a rigid body

$$\mathbf{v}_P = \mathbf{v_o} + \boldsymbol{\omega} \times \mathbf{r}_{P/O} = -5\mathbf{k} \times (3\cos 10°\,\mathbf{i} - 3\sin 10°\,\mathbf{j}) = -15\sin 10°\,\mathbf{i} - 15\cos 10°\,\mathbf{j}$$
$$= -2.604\,\mathbf{i} - 14.772\,\mathbf{j}$$

and

$$\mathbf{a}_P = \mathbf{a}_O + \boldsymbol{\omega}\,\mathbf{x}(\boldsymbol{\omega}\mathbf{x}\mathbf{r}_{P/O}) + \boldsymbol{\alpha}\mathbf{x}\mathbf{r}_{P/O}$$

$$\mathbf{a}_P = (-66.5\mathbf{i} + 54.3\mathbf{j})\ \frac{\text{m}}{\text{s}^2}$$

$$|\mathbf{a}_P| = 85.9\,\frac{\text{m}}{\text{s}^2}$$

Alternate solution: The bar is rotating about a fixed point. Thus any point on the bar moves on a circular arc about the point of support. The particle P has two components of acceleration, one directed between P and O (the normal acceleration), and one tangent to the path of P whose direction is determined using the right hand rule (the tangential component).

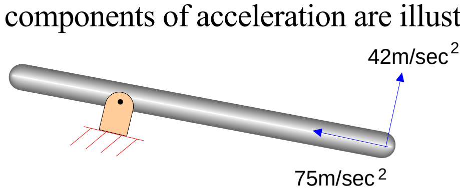
The component normal to the path of P is

$$a_n = 3\text{m}(5\,\frac{\text{rad}}{\text{s}})^2 = 75\,\frac{\text{m}}{\text{s}^2}$$

and is directed between P and O. The tangential acceleration is

$$a_t = (3\text{m})(14\,\frac{\text{rad}}{\text{s}^2}) = 42\,\frac{\text{m}}{\text{s}^2}$$

The normal and tangential components of acceleration are illustrated on the diagram below.



Problem 1.3 illustrates the use of the relative acceleration equation of rigid body kinetics.

1.4 At t = 0, a particle of mass 1.2 kg is traveling with a speed of 10 m/s that is increasing at a rate of 0.5 m/s^2. The local radius of curvature at this instant is 50 m. After the particle travels 100 m, the radius of curvature of the particle's path is 50 m.

(a) What is the speed of the particle after it travels 100 m?
(b) What is the magnitude of the particle's acceleration after it travels 100 m?
(c) How long does it take the particle to travel 100 m?
(d) What is the external force acting on the particle after it travels 100 m?

Given: m = 1.2 kg, v(t=0) = 10 m/s, dv/dt= 0.5 m/s^2, and r = 25 m when s = 100 m
Find: (a) v when s = 100 m, (b) a when s = 3 m, (c) t when s = 3 m

Solution: Let s(t) be the displacement of the particle, measured from t = 0. The particle's velocity is

$$v(t) = \int_0^t \frac{dv}{dt}\,dt + v(0) = \int_0^t 0.5\,dt = 0.5t + 10$$

By definition v=ds/dt. Thus the displacement of the particle is obtained as

$$s(t) = \int_0^t v\,dt + s(0) = \int_0^t \left(0.5t + 10\right) dt = 0.25t^2 + 10t$$

When s = 100 m,

$$100 \text{ m} = 0.25t^2 + 10t \Rightarrow t = 8.28 \text{ s}$$

(a) The velocity when s = 100 m is

$$v = 0.5(8.28) + 10 = 14.14 \text{ m/s}$$

(b) Since the particle is traveling along a curved path, its acceleration has two components: a tangential component equal to the rate of change of the velocity

$$a_t = \frac{dv}{dt} = 0.5 \text{ m/s}^2$$

and a normal component directed toward the center of curvature

$$a_n = \frac{v^2}{r} = \frac{(14.14 \text{ m/s})^2}{50 \text{ m}} = 4.00 \text{ m/s}^2$$

The magnitude of the acceleration at this instant is

$$|\mathrm{a}| = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.5\ \mathrm{m/s^2})^2 + (4.00\ \mathrm{m/s^2})^2}$$
$$|\mathrm{a}| = 4.03\ \mathrm{m/s^2}$$

(c) The time for the particle to travel 100 m is previously calculated as $t = 8.28$ s
(d) The external force equation written in terms of magnitudes is

$$\sum |\mathbf{F}| = m|a|$$

which upon application to the particle gives

$$\sum |\mathbf{F}| = (1.2\ \mathrm{kg})\left(4.03\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) = 4.84\ \mathrm{N}$$

Problem 1.4 illustrates the kinematics of a particle traveling along a curved path.

1.5 The machine of Figure P1.15 has a vertical displacement, $x(t)$. The machine has component which rotates with a constant angular speed, ω. The center of mass of the rotating component is a distance e from its axis of rotation. The center of mass of the rotating component is as shown at $t = 0$. Determine the vertical component of the acceleration of the rotating component.

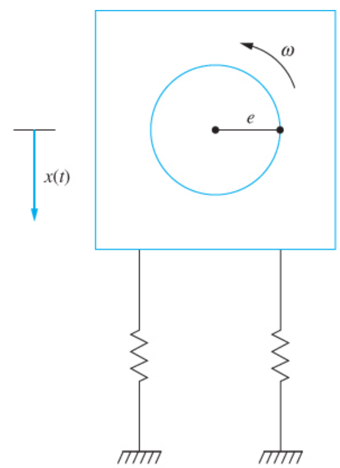


FIGURE P1.5

Given: e, ω, x (t)

Find: a_y

Solution: The particle of interest is on a component that moves relative to the machine. From the relative acceleration equation,

$$\mathbf{a}_G = \mathbf{a}_M + \mathbf{a}_{G/M}$$

where

$$\mathbf{a}_M = -\ddot{x}(t)\,\mathbf{j}$$

and

$$\mathbf{a}_{G/M} = e\omega^2\left(-\cos\theta\mathbf{i} - \sin\theta\mathbf{j}\right)$$

Since the angular velocity of the rotating component is constant and θ = 0 when t = 0,

$$\theta = \omega t$$

Hence the vertical acceleration of the center of mass of the rotating component is

$$a_y = -\ddot{x}(t) - e\omega^2 \sin \omega t$$

Problem 1.5 illustrates application of the relative acceleration equation. Vibrations of machines subject to a rotating unbalance are considered in Chapter 4.

1.6 The rotor of Figure P1.6 consists of a disk mounted on a shaft. Unfortunately, the disk is unbalanced, and the center of mass is a distance e from the center of the shaft. As the disk rotates, this causes a phenomena called "whirl", where the disk bows. Let r be the instantaneous distance from the center of the shaft to the original axis of the shaft and θ be the angle made by a given radius with the horizontal. Determine the acceleration of the mass center of the disk.

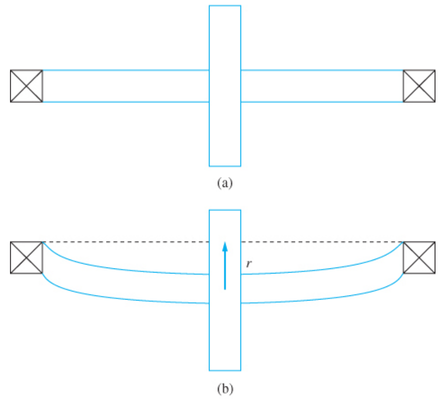


FIGURE P1.6

Given: e, r

Find: $\bar{\mathbf{a}}$

Solution: The position vector from the origin to the center of the disk is $r\mathbf{i}_r$ where r varies with time. The mass center moves in a circular path about the center of the disk. The relative acceleration equation gives

$$\mathbf{a}_c = \mathbf{a}_o + \boldsymbol{\alpha} \times r\mathbf{i}_r + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times r\mathbf{i}_r) + \ddot{r}\mathbf{i}_r + 2\boldsymbol{\omega} \times \dot{r}\mathbf{i}_r$$

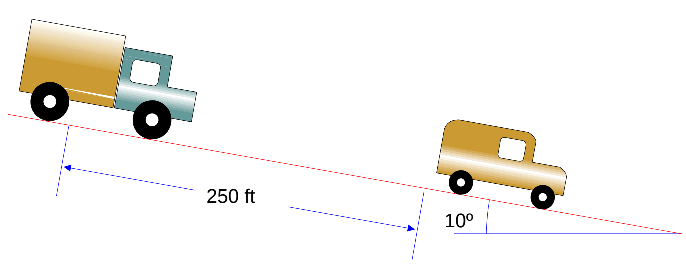
$$\mathbf{a}_c = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{i}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{i}_\theta$$

The acceleration of the mass center is then

$$\bar{\mathbf{a}} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{i}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\mathbf{i}_\theta - e\omega^2[\cos(\omega t - \theta)\,\mathbf{i}_r + \sin(\omega t - \theta)\,\mathbf{i}_\theta]$$

Problem 1.6 illustrates application of the relative acceleration equation.

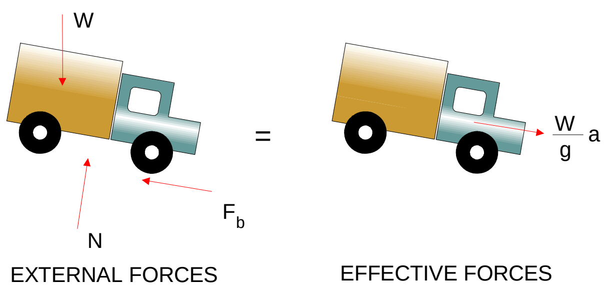
1.7 A 2 ton truck is traveling down an icy, 10º hill at 50 mph when the driver sees a car stalled at the bottom of the hill 250 ft away. As soon as he sees the stalled car, the driver applies his brakes, but due to the icy conditions, a braking force of only 2000 N is generated. Does the truck stop before hitting the car?



Given: W = 4000 lb., $\theta = 10^{\circ}$, d = 250 ft., F_b = 2000 N = 449.6 lb,
v_o = 50 mph = 73.33 ft/sec

Find: v = 0 before x = 250 ft.

Solution: Application of Newton's Law to the free body diagram of the truck at an arbitrary instant



$$\left(\sum F_x\right)_{ext.} = \left(\sum F_x\right)_{eff.}$$

$$-F_b + W\sin\theta = \frac{W}{g}a$$

$$a = g\left(-\frac{F_b}{W} + \sin\theta\right)$$

$$a = 32.2\ \frac{\text{ft}}{\sec^2}\left(-\frac{449.6\ \text{lb}}{4000\ \text{lb}} + \sin 10^0\right)$$

$$a = 1.973\ \frac{\text{ft}}{\sec^2}$$

Since the acceleration is constant, the velocity and displacement of the truck are

$$v = at + v_0 = 1.973t + 73.33$$

$$x = a\frac{t^2}{2} + v_0 t = 0.986t^2 + 73.33t$$

The acceleration is positive thus the vehicle speeds up as it travels down the incline. The truck does not stop before hitting the car.

Problem 1.7 illustrates application of Newton's Law to a particle and kinematics of constant acceleration.

1.8 The contour of a bumpy road is approximated by $y(x) = 0.03\sin(0.125x)$ m. What is the amplitude of the vertical acceleration of the wheels of an automobile as it travels over this road at a constant horizontal speed of 40 m/s?

Given: $y(x) = 0.03\sin(0.125x)$ m, $v = 40$ m/s

Find: A

Solution: Since the vehicle is traveling at a constant horizontal speed its horizontal distance traveled in a time t is $x = vt$. Thus the vertical displacement of the vehicle is

$$y(t) = 0.03\sin\left[0.125(40t)\right] = 0.03\sin(5t)\ \text{m}$$

The vertical velocity and acceleration of the vehicle are calculated as

$$v(t) = 0.03(5)\cos(5t) = 0.15\cos(5t)\ \text{m/s}$$
$$a(t) = -0.15(5)\sin(5t) = -0.75\sin(5t)\ \text{m/s}^2$$

Thus the amplitude of acceleration is A=0.75 m/s^2.

Problem 1.8 illustrates the relationship between displacement, velocity, and acceleration for the motion of a particle.

1.9 The helicopter of Figure P1.9 has a horizontal speed of 110 ft/s and a horizontal acceleration of 3.1 ft/s^2. The main blades rotate at a constant speed of 135 rpm. At the instant shown, determine the velocity and acceleration of particle *A*.

Given: $v_h = 110$ ft/s, a_h=3ft/s^2, $\omega = 135$ rpm = 14.1 rad/s, r = 2.1 ft

Find: $\mathbf{v_A}$, $\mathbf{a_A}$

Solution: Construct a x-y coordinate system in the horizontal plane

As illustrated. Using this coordinate system

$$\mathbf{v} = -110\mathbf{i}\ \text{ft/s},\ \ \mathbf{a} = -3\mathbf{i}\ \text{ft/s}^2$$

The position vector of A relative to the helicopter at this instant is

$$\mathbf{r_{A/h}} = r\left[\cos(\pi/4)\mathbf{i} - \sin(\pi/4)\mathbf{j}\right] = 1.48\mathbf{i} - 1.48\mathbf{j}$$

The relative velocity equation is used to determine the velocity of particle A as

$$\mathbf{v_A} = \mathbf{v_h} + \omega\mathbf{k}\times\mathbf{r_{A/h}}$$
$$\mathbf{v_A} = -110\mathbf{i} + 14.1\mathbf{k}\times(1.48\mathbf{i} - 1.48\mathbf{j})$$
$$\mathbf{v_A} = -89.1\mathbf{i} + 20.9\mathbf{j}\ \text{ft/s}$$

The relative acceleration equation is used to determine the acceleration of particle A as

$$\mathbf{a}_A = \mathbf{a}_h + \alpha\mathbf{k} \times \mathbf{r}_{A/h} + \omega\mathbf{k} \times (\omega\mathbf{k} \times \mathbf{r}_{A/h})$$
$$\mathbf{a}_A = -3.1\mathbf{i} + 14.1\mathbf{k} \times (20.87\mathbf{i} + 20.87\mathbf{j})$$
$$\mathbf{a}_A = -297.4\mathbf{i} + 294.6\mathbf{j} \text{ ft/s}^2$$

Problem 1.9 illustrates the use of the relative velocity and relative acceleration equations.

1.10 For the system shown in Figure P1.10, the angular displacement of the thin disk is given by $\theta(t) = 0.03\sin\left(30t + \frac{\pi}{4}\right)$ rad. The disk rolls without slipping on the surface. Determine the following as functions of time. (a) The acceleration of the center of the disk. (b) The acceleration of the point of contact between the disk and the surface. (c) The angular acceleration of the bar. (d) The vertical displacement of the block. (Hint: Assume small angular oscillations ϕ of the bar. Then $\sin\phi \approx \phi$.)

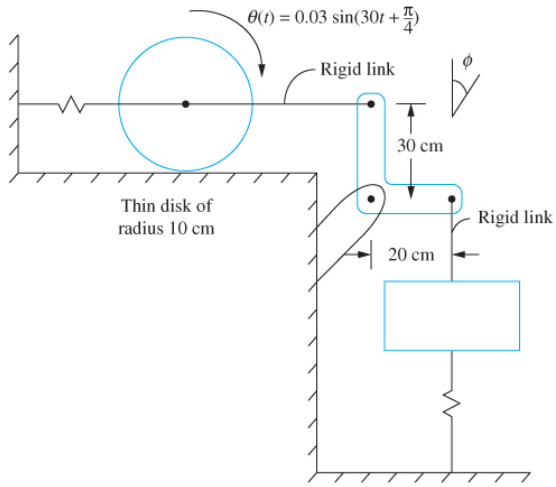


FIGURE P1.10

Given: $\theta(t)$, $r_d = 0.1$ m, $\ell = 0.3$ m, $d = 0.2$ m

Find: (a) $\bar{a}_d$ (b) $\bar{a}_c$ (c) α (d) x

Solution: (a) The angular acceleration of the disk is

$$\ddot{\theta}(t) = -(30)^2 0.03\sin\left(30t + \frac{\pi}{4}\right) = -27\sin\left(30t + \frac{\pi}{4}\right)\frac{\text{rad}}{\text{s}^2}$$

Since the disk rolls without slip the acceleration of the mass center is

$$a_d = r\alpha = (0.1\text{ m})\left(-27\sin\left(30t + \frac{\pi}{4}\right)\frac{\text{rad}}{\text{s}^2}\right) = -2.7\sin\left(30t + \frac{\pi}{4}\right)\frac{\text{m}}{\text{s}^2}$$

(b) Since the disk rolls without slip the horizontal acceleration of the point of contact is zero. The vertical acceleration is $r\dot{\theta}^2$ towards the center

$$\mathbf{a}_c = (0.1\text{ m})\left[(30)(0.03)\cos\left(30t + \frac{\pi}{4}\right)\frac{\text{rad}}{\text{s}}\right]^2\mathbf{j}$$

$$= 0.081\left[\cos\left(30t+\frac{\pi}{4}\right)\right]^2 \frac{\text{m}}{\text{s}^2}\,\mathbf{j}$$

(c) Assuming small ϕ the angular displacement of the link is $\ell\phi = x_d$ or

$$\ddot{\phi} = \frac{\ddot{x}}{\ell} = \frac{-2.7\sin\left(30t+\frac{\pi}{4}\right)\frac{\text{m}}{\text{s}^2}}{0.3\ \text{m}}$$

$$\ddot{\phi} = -9\sin\left(30t+\frac{\pi}{4}\right)\frac{\text{r}}{\text{s}^2}$$

(d) The displacement of the mass center of the block is $x = d\phi$ or

$$x = (0.2\,)\frac{9}{(30)^2}\sin\left(30t+\frac{\pi}{4}\right) = 2\ \sin\left(30t+\frac{\pi}{4}\right)\ \text{mm}$$

Problem 1.10 illustrates angular acceleration and acceleration of a body rolling without slip.

1.11 The velocity of the block of the system of Figure P1.11 is $\dot{y} = 0.02\sin 20t$ m/s downward. (a) What is the clockwise angular displacement of the pulley? (b) What is the displacement of the cart?

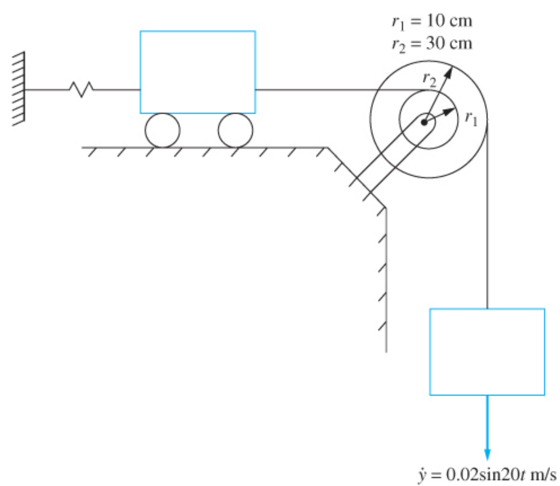


FIGURE P1.11

Given: $\dot{y}$, $r_1 = 0.1$ m, $r_2 = 0.3$ m

Find: (a) $\theta(t)$ (b) $x(t)$

Solution: the displacement of the block is

$$y(t) = \int \dot{y}\,dt = 0.001(1-\cos 20t)\ \text{m}$$

(a) The angular displacement of the pulley is

$$\theta = \frac{y}{r_2} = \frac{0.001(1-\cos 20t)\ \text{m m}}{0.3\ \text{m}} = 3.33\times 10^{-4}(1-\cos 20t)\ \text{rad}$$

(b) The displacement of the cart is

$$x = \frac{r_1}{r_2}y = \frac{0.1\,m}{0.3\,m}[0.001(1-\cos 20t)\ \text{m}] = 3.33\times 10^{-4}(1-\cos 20t)\ \text{m}$$

Problem 1.11 illustrates velocity and kinematics.

1.12 A 60-lb block is connected by an inextensible cable through the pulley to the fixed surface, as shown in Figure P1.12. A 40-lb weight is attached to the pulley, which is free to move vertically. A force of magnitude $P = 100(1+ e^{-t})$ lb tows the block. The system is released from rest at $t = 0$.

(a) What is the acceleration of the 60 lb block as a function of time?

(b) How far does the block travel up the incline before it reaches a velocity of 2 ft/sec?

Given: $W_1 = 60$ lbs, $W_2 = 40$ lbs, $P = 100(1+e^{-t})$ lb, $\mu = 0.3$, $\theta = 45°$

Find: a(t), x(v = 2 ft/sec)

Solution: Let x be the distance the block travels from t = 0. Let y be the vertical distance traveled by the pulley from t = 0. The total length of the cable connecting the block, the pulley and the surface is constant as the block moves up the incline. Thus, referring to the diagrams below. At t = 0, $\ell = a + b + c$. At an arbitrary time, $\ell = a + x + b - y + c - y = a + b + c + x - 2y$. Hence $y = x/2$.

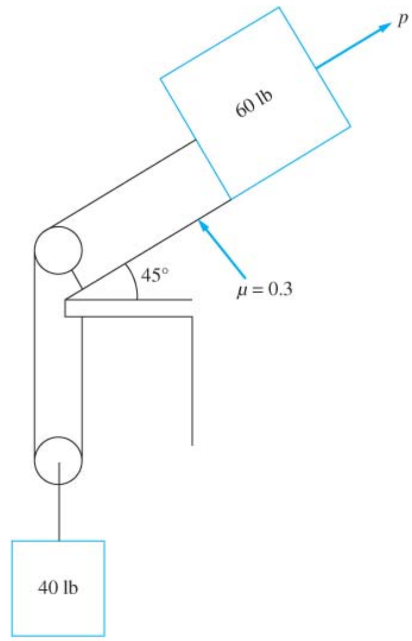
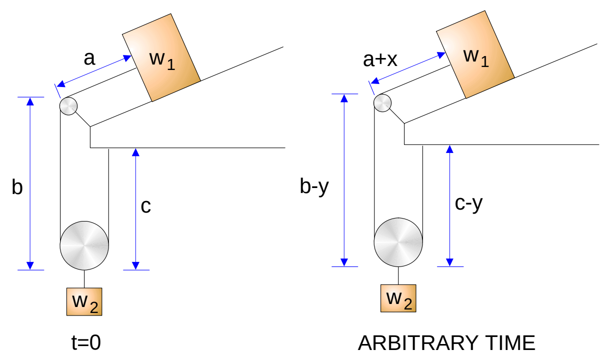
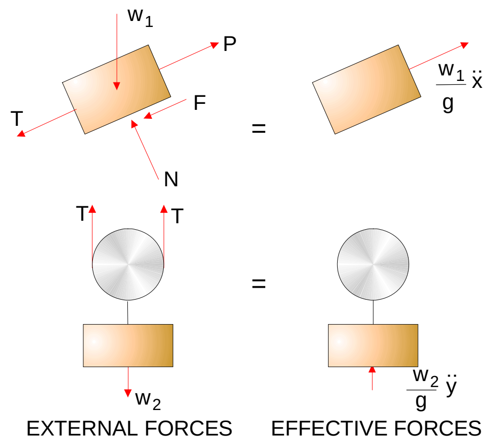


FIGURE P1.12



Free body diagrams of the blocks are shown at an arbitrary instant of time.



From the free body diagrams of the pulley

$$\left(\sum F\right)_{ext.} = \left(\sum F\right)_{eff.}$$
$$2T - m_2 g = m_2 \ddot{y} \qquad (1)$$
$$T = \frac{m_2}{2}\left(\ddot{y} + g\right)$$

Summation of forces in the direction normal to the incline for the block yields

$$N = m_1 g \cos\theta \qquad (2)$$

Summing forces in the direction along the incline on the block

$$\left(\sum F\right)_{ext.} = \left(\sum F\right)_{eff.}$$
$$-T + P - F - m_1 g \sin\theta = m_1 \ddot{x} \qquad (3)$$

Noting that $F = \mu N$ and using eqs. (1) and (2) in eq. (3) gives

$$\ddot{x} = \frac{-\frac{m_2 g}{2} + P - \mu m_1 g \cos\theta - m_1 g \sin\theta}{m_1 + \frac{m_2}{4}} \qquad (4)$$

Substituting given values leads to

$$\ddot{x} = 11.42 + 46.0e^{-t} \ \frac{\text{ft}}{\text{s}^2}$$

The velocity is calculated from

$$\int_0^v dv = \int_0^t \left(11.42 + 46.9e^{-t}\right)\, dt$$
$$v = 46.0 + 11.42t - 46.0e^{-t} \tag{5}$$

Setting v = 2ft/sec in eq. (5) and solving the resulting equation for t by trial and error reveals that it takes 0.0354 sec for the velocity to reach 2 ft/sec. The displacement from the initial position is calculated from

$$\int_0^x dx = \int_0^t \left(46.0 + 11.42t - 46.0e^{-t}\right) dt$$
$$x(t) = -46.0 + 46.0t + 5.71t^2 + 46.0e^{-t} \tag{6}$$

Setting t = 0.0354 sec in eq.(6), leads to

$$x = 0.0356 \text{ ft}$$

Problem 1.12 illustrates the application of Newton's Law to a particle, the kinematics of pulley systems, and relationships between acceleration, velocity, and displacement. Note that the time required to attain a velocity of 2 ft/sec could have been attained using impulse and momentum.

1.13 Repeat Problem 1.12 for a force of $P = 100t$ N.

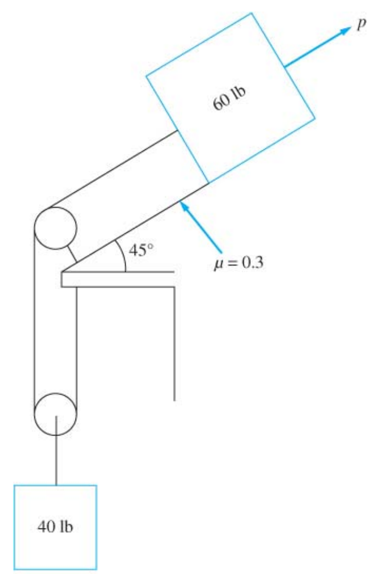
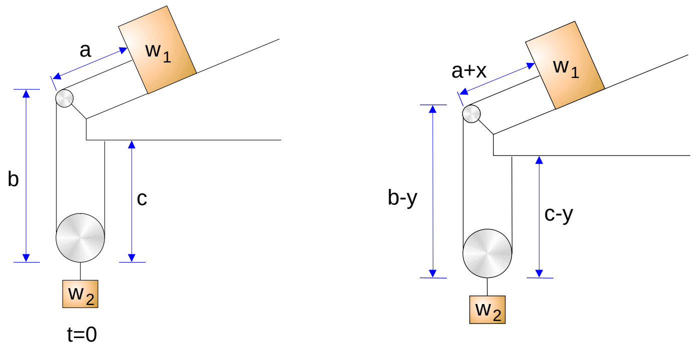


FIGURE P1.12

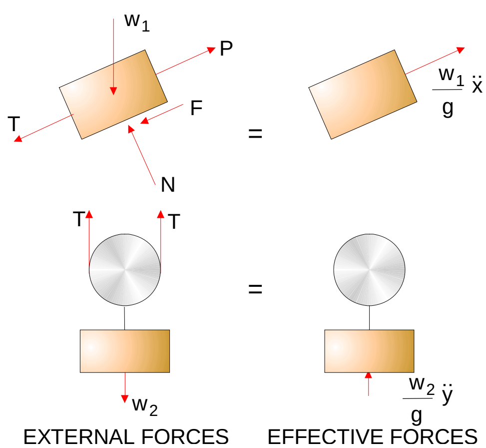
Given: W_1 = 60 lbs, W_2 = 40 lbs, P = 100t lbs, $\mu = 0.3$, θ = 45º

Find: a(t), x(v = 2 ft/sec)

Solution: Let x be the distance the block travels from t = 0. Let y be the vertical distance traveled by the pulley from t = 0. The total length of the cable connecting the block, the pulley and the surface is constant as the block moves up the incline. Thus, referring to the diagrams below. At t = 0, ℓ = a + b + c. At an arbitrary time, ℓ = a + x + b – y + c – y = a + b + c + x – 2y. Hence y = x/2.



Free body diagrams of the blocks are shown at an arbitrary instant of time.



From the free body diagrams of the pulley

$$\left(\sum F\right)_{ext.} = \left(\sum F\right)_{eff.}$$

$$2T - m_2 g = m_2 \ddot{y} \qquad (1)$$

$$T = \frac{m_2}{2}\left(\ddot{y} + g\right)$$

Summation of forces in the direction normal to the incline for the block yields

$$N = m_1 g \cos\theta \qquad (2)$$

Summing forces in the direction along the incline on the block

$$\left(\sum F\right)_{ext.} = \left(\sum F\right)_{eff.}$$

$$-T + P - F - m_1 g \sin\theta = m_1 \ddot{x} \qquad (3)$$

Noting that F = μN and using eqs.(1) and (2) in eq.(3) gives

$$\ddot{x} = \frac{-\dfrac{m_2 g}{2} + P - \mu m_1 g \cos\theta - m_1 g \sin\theta}{m_1 + \dfrac{m_2}{4}} \tag{4}$$

Substituting given values leads to

$$\ddot{x} = 36.79t - 34.57$$

Note that the acceleration is initially negative, then becomes positive.

$$\int_o^v dv = \int_o^t \left(36.79t - 34.57\right)dt \tag{5}$$
$$v = 18.40t^2 - 34.57t$$

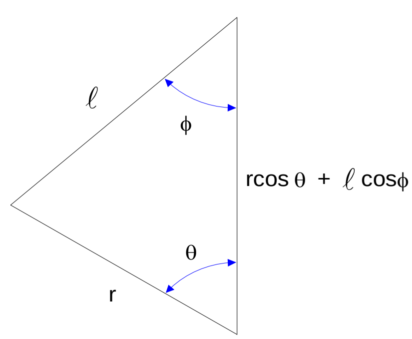
Setting v = 2 ft/sec in eq.(5) and solving the resulting quadratic equation reveals that it takes 2.07 sec for the velocity to reach 2 ft/sec. The displacement from the initial position is calculated from

$$\int_o^x dx = \int_o^t \left(18.4t^2 - 34.57t\right)dt \tag{6}$$

$$x(t) = 6.13t^3 - 17.28t^2$$
$$x(2.07\,\text{sec}) = -19.76\,\text{ft}$$

Problem 1.13 illustrates the application of Newton's Law to a particle, the kinematics of pulley systems, and relationships between acceleration, velocity, and displacement. Note that the time required to attain a velocity of 2 ft/sec could have been attained using impulse and momentum.

1.14 Figure P1.14 shows a schematic diagram of a one-cylinder reciprocating one-cylinder engine. If at the instant time shown the piston has a velocity *v* and an acceleration *a*, determine (a) the angular velocity of the crank and (b) the angular acceleration of the crank in terms of *v*, *a*, the crank radius *r*, the connecting rod length ℓ, and the crank angle θ.



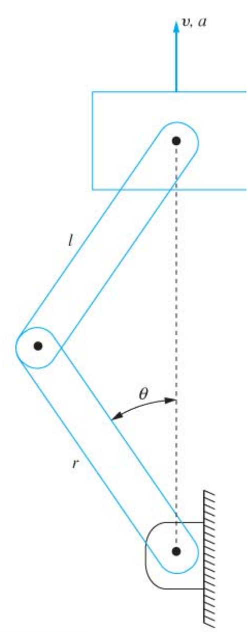


FIGURE P1.14

Given: r, ℓ, θ, v, a

Find: α_{AB}

Solution: (a) From the law of sines

$$\frac{r}{\sin\phi}=\frac{\ell}{\sin\theta}$$

or

$$\sin\phi=\frac{r}{\ell}\sin\theta \qquad (1)$$

Then from trigonometry

$$\begin{aligned}\cos\phi &= \sqrt{1-\sin^2\phi}\\ &= \sqrt{1-\left(\frac{r}{\ell}\sin\theta\right)^2}\end{aligned} \qquad (2)$$

Using the relative velocity equation,

$$\begin{aligned}\vec{v}_B &= \vec{v}_A+\vec{\omega}_{AB}x\vec{r}_{B/A}\\ &= \omega_{AB}\vec{k}x\left(-r\sin\theta\vec{i}+r\cos\theta\vec{j}\right)\\ &= -r\omega_{AB}\cos\theta\vec{i}-r\omega_{AB}\sin\theta\vec{j}\end{aligned}$$

and

$$\begin{aligned}\vec{v}_C &= v\vec{j}=\vec{v}_B+\vec{\omega}_{BC}x\vec{r}_{C/B}\\ &= \vec{v}_B+\omega_{BC}\vec{k}x\left(\ell\sin\phi\vec{i}+\ell\cos\phi\vec{j}\right)\\ &= \left(-r\omega_{AB}\sin\theta+\ell\omega_{BC}\sin\phi\right)\vec{j}-\left(r\omega_{AB}\cos\theta+\ell\omega_{BC}\cos\phi\right)\vec{i}\end{aligned}$$

The x component yields

$$\omega_{BC}=-\frac{r}{\ell}\omega_{AB}\frac{\cos\theta}{\cos\phi} \qquad (3)$$

which when substituted into the y component leads to

$$\omega_{AB}=-\frac{v}{r(\sin\theta+\cos\theta\tan\phi)} \qquad (4)$$

(b)The relative acceleration equations give

$$\vec{a}_B = a_A + \alpha_{AB} x \vec{r}_{B/A} + \vec{\omega}_{AB} x \left(\vec{\omega}_{AB} x \vec{r}_{B/A}\right)$$
$$= \left(-r\alpha_{AB}\cos\theta + r\omega_{AB}^2 \sin\theta\right)\vec{i} + \left(-r\alpha_{AB}\sin\theta - r\omega_{AB}^2\cos\theta\right)\vec{j}$$

and

$$\vec{a}_C = a\vec{j} = \vec{a}_B + \vec{\alpha}_{BC} x r_{C/B} + \vec{\omega}_{BC} x \vec{\omega}_{BC} x \vec{r}_{CB}$$
$$= \left(-r\alpha_{AB}\cos\theta - r\omega_{AB}^2\sin\theta - \ell\alpha_{BC}\cos\phi - \ell\omega_{BC}^2\sin\phi\right)\vec{i}$$
$$+ \left(-r\alpha_{AB}\sin\theta + r\omega_{AB}^2\cos\theta + \ell\alpha_{BC}\sin\phi - \ell\omega_{BC}^2\cos\phi\right)\vec{j}$$

The x component is used to determine

$$\alpha_{BC} = -\frac{1}{\ell\cos\phi}\left(r\omega_{AB}^2\sin\theta + r\alpha_{AB}\cos\theta + \ell\omega_{BC}^2\sin\phi\right)$$

Which when used in the y component leads to

$$\alpha_{AB} = -\frac{a - r\omega_{AB}^2\cos\theta + \ell\omega_{BC}^2\cos\phi - r\omega_{AB}^2\sin\theta\tan\phi + \ell\omega_{BC}^2\sin\phi\tan\phi}{r\left(\sin\theta + \cos\theta\tan\phi\right)} \qquad (5)$$

Equation (5) is used to determine the angular acceleration of the crank using eqs.(1) - (4).

Problem 1.14 illustrates application of the relative velocity and relative acceleration equations for rigid body kinematics.

1.15 Determine the reactions at *A* for the two-link mechanism of Figure P1.15. The roller at C rolls on a frictionless surface.

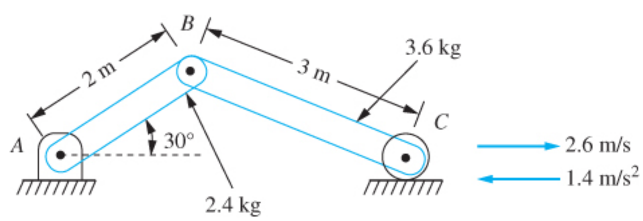


FIGURE P1.15

Given : $\theta = 30°$, $L_{AB} = 2$ m, $L_{BC} = 3$ m, $m_{AB} = 2.4$ kg, $m_{BC} = 3.6$ kg, $v_C = 2.6$ m/sec, $a_C = -1.4$ m/sec^2

Find : A_x , A_y

Solution : From the law of sines

$$\frac{\sin\theta}{L_{BC}} = \frac{\sin\phi}{L_{AB}}$$
$$\sin\phi = \frac{L_{AB}}{L_{BC}}\sin\theta = 0.333$$

From trigonometry

$$\cos\phi = \sqrt{1-\sin^2\phi} = 0.943$$

The relative position vectors are

$$\vec{r}_{B/A} = L_{AB}\left(\cos\theta\vec{i} + \sin\theta\vec{j}\right) = 1.73\vec{i} + \vec{j}\ m$$
$$\vec{r}_{C/B} = L_{BC}\left(\cos\phi\vec{i} - \sin\phi\vec{j}\right) = 2.83\vec{i} - \vec{j}\ m$$

Using the relative velocity equation between two particles on a rigid body,

$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AB}\mathbf{kxr}_{B/A}$$
$$\mathbf{v}_B = -\omega_{AB}\mathbf{i} + 1.73\,\omega_{AB}\mathbf{j}$$
$$\mathbf{v}_C = \mathbf{v}_{BC} + \omega_{BC}\mathbf{kxr}_{C/B}$$
$$2.6\,\mathbf{i} = \left(-\omega_{AB} + \omega_{BC}\right)\mathbf{i} + \left(1.73\,\omega_{AB} + 2.83\omega_{BC}\right)\mathbf{j}$$

Equating like components from both sides leads to

$$1.73\,\omega_{AB} + 2.83\omega_{BC} = 0$$
$$-\omega_{AB} + \omega_{BC} = 2.6$$

Simultaneous solution of the above equations leads to

$$\omega_{AB} = -1.61\frac{\text{rad}}{\text{s}},\ \omega_{BC} = 0.986\frac{\text{rad}}{\text{s}}$$

Use of the relative acceleration equation between two particles on a rigid body,

$$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB}\mathbf{kxr}_{B/A} + \omega_{AB}\mathbf{k}x\left(\omega_{AB}\mathbf{kxr}_{B/A}\right)$$
$$\mathbf{a}_B = \left(-\alpha_{AB} - 4.48\right)\mathbf{i} + \left(1.73\alpha_{AB} - 2.59\right)\mathbf{j}\frac{\text{m}}{\text{s}^2}$$
$$\mathbf{a}_C = \mathbf{a}_B + \alpha_{BC}\mathbf{kxr}_{C/B} + \omega_{BC}\mathbf{kx}\left(\omega_{BC}\mathbf{kxr}_{C/B}\right)$$
$$-1.4\mathbf{i} = \left(-\alpha_{AB} + \alpha_{BC} + 7.23\right)\mathbf{i} + \left(1.72\alpha_{AB} + 2.83\alpha_{BC} - 1.62\right)\mathbf{j}$$

Equating like components from both sides leads to

$$1.72\alpha_{AB} + 2.83\alpha_{BC} = 1.62$$
$$-\alpha_{AB} + \alpha_{BC} = -8.63$$

Simultaneous solution of the above equations leads to

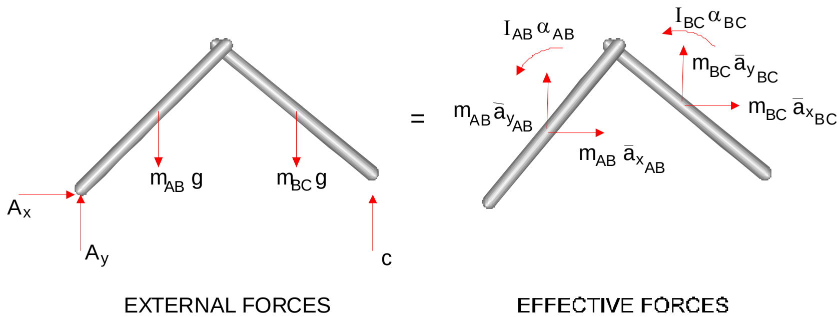
$$\alpha_{AB} = 5.72\frac{\text{rad}}{\text{s}^2}, \alpha_{\mathbf{B}C} = -2.91\frac{\text{rad}}{\text{s}^2}$$

The relative acceleration equations are used to calculate the accelerations of the mass centers of the links as

$$\mathbf{a}_{AB} = -5.09\mathbf{i} + 3.65\mathbf{j}\frac{\text{m}}{\text{s}^2}$$

$$\mathbf{a}_{BC} = -13.64\mathbf{i} + 3.66\mathbf{j}\frac{\text{m}}{\text{s}^2}$$

Free body diagrams of the two bar linkage at this instant of time are shown below



Summing moments about C

$$\left(\sum M_C\right)_{ext.} = \left(\sum M_C\right)_{eff.}$$

$$A_y\left(L_{AB}\cos\theta + L_{BC}\cos\phi\right) - m_{AB}g\left(\frac{L_{AB}}{2}\cos\theta + L_{BC}\cos\phi\right) - m_{BC}g\frac{L_{BC}}{2}\cos\phi$$

$$= -\bar{I}_{AB}\alpha_{AB} - \bar{I}_{BC}\alpha_{BC} + m_{AB}\bar{a}_{x_{AB}}\frac{L_{AB}}{2}\sin\theta + m_{BC}\bar{a}_{x_{BC}}\frac{L_{BC}}{2}\sin\phi$$

$$+ m_{AB}\bar{a}_{y_{AB}}\left(\frac{L_{AB}}{2}\cos\theta + L_{BC}\cos\phi\right) + m_{BC}\bar{a}_{y_{BC}}\frac{L_{BC}}{2}\cos\phi$$

Noting that

$$\bar{I}_{AB} = \frac{1}{12}m_{AB}L_{AB}^2 = 0.8\,\text{kg}\cdot\text{m}^2, \bar{I}_{BC} = \frac{1}{12}m_{BC}L_{BC}^2 = 2.7\,\text{kg}\cdot\text{m}^2$$

and substituting given and calculated values and solving for A_y leads to

$$A_y = 28.49\,\text{N}$$

Summing forces in the horizontal direction

$$\left(\sum F_x\right)_{ext.} = \left(\sum F_x\right)_{eff.}$$
$$A_x = m_{AB}\bar{a}_{x_{AB}} + m_{BC}\bar{a}_{x_{BC}}$$

Substituting given and calculated values leads to

$$A_x = -61.32\,\text{N}$$

Problem 1.15 illustrates (a) application of the relative velocity equation for a linkage, (b) application of the relative acceleration equation for a linkage, and (c) application of Newton's laws to a system of rigid bodies. This problem is a good illustration of the effectiveness of the effective force method of application of Newton's Laws. Use of this method allows a free body diagram of the entire linkage to be drawn and used to solve for the unknown reactions. Application of Newton's Laws to a single rigid body exposes the reactions in the pin connection at B and complicates the solution.

1.16 Determine the angular acceleration of each of the disks in Figure P1.16.

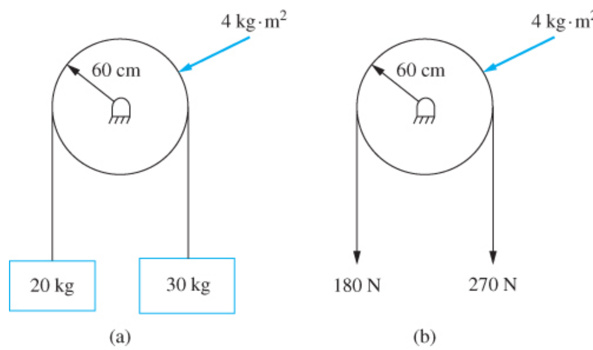


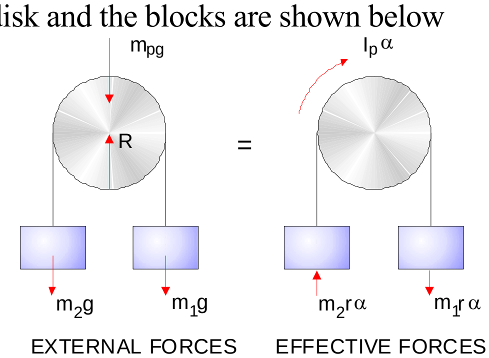
FIGURE P1.16

Given: Disk of $I_P = 4$ kg-m^2, r = 60 cm with (a) $m_1 = 30$ kg and $m_2 = 20$ kg blocks attached or (b) $F_1 = 270$ N and $F_2 = 180$ N forces attached.

Find: α

Solution:

(a) Free body diagrams of the disk and the blocks are shown below



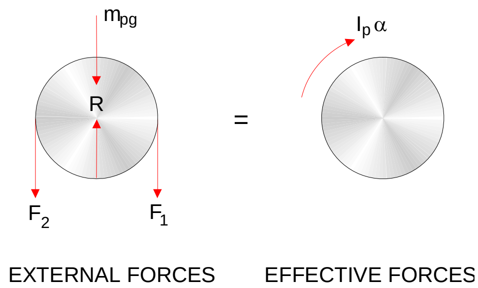
Summing moments about the center of the disk

$$(\Sigma M_O)_{ext.} = (\Sigma M_O)_{eff.}$$

$$m_1 gr - m_2 gr = I_p \alpha + m_1 r^2 \alpha + m_2 r^2 \alpha$$

$$\alpha = \frac{(m_1 - m_2)gr}{I_p + (m_1 + m_2)r^2} = 2.68 \frac{\text{rad}}{\text{s}^2}$$

(b) Free body diagrams of the disk are shown below



Summing moments about the center of the disk

$$(\Sigma M_O)_{ext.} = (\Sigma M_O)_{eff.}$$

$$F_1 r - F_2 r = I_p \alpha$$

$$\alpha = \frac{(F_1 - F_2)r}{I_P} = 13.5 \frac{\text{rad}}{\text{s}^2}$$

Problem 1.16 illustrates application of Newton's Laws to systems of rigid bodies. It also illustrates the difference between an applied force and a mass.

1.17 Determine the reactions at the pin support and the applied moment if the bar of Figure P1.17 has a mass of 50 g.

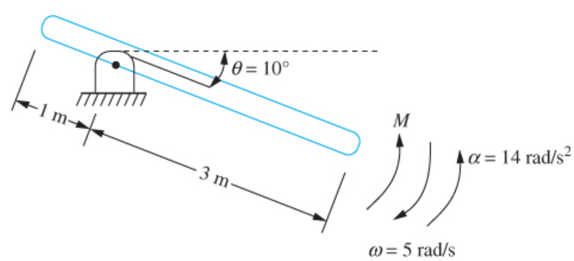


FIGURE P1.17

Given: $\alpha = 14$ rad/sec^2, $\omega = -5$ rad/sec, m = 50 kg

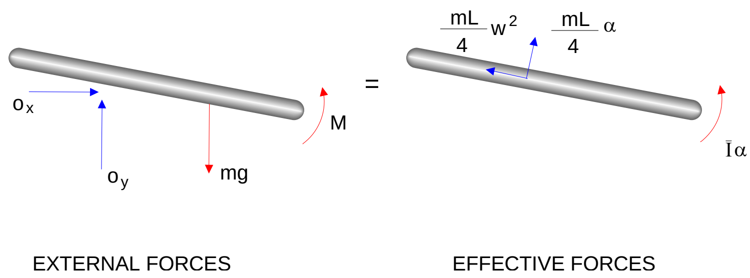
L = 4 m, $\theta = 10°$

Find: M, O_x, O_y

Solution: The bar's centroidal moment of inertia of the bar

$$\bar{I} = \frac{1}{12} m L^2 = \frac{1}{12}(50\text{ kg})(4\text{ m})^2 = 66.67\text{ kg}\cdot\text{m}^2$$

Free body diagrams of the bar at this instant are shown below



Summing moments about O

$$(\sum M_O)_{ext.} = (\sum M_O)_{eff.}$$

$$M - mg\frac{L}{4}\cos\theta = \bar{I}\alpha + m\frac{L}{4}\alpha\frac{L}{4}$$

$$M = (\bar{I} + m\frac{L^2}{16})\alpha + mg\frac{L}{4}\cos\theta$$

$$= [66.67\ \text{kg - m}^2 + \frac{(50\ \text{kg})(4\ \text{m})^2}{16}](14\frac{\text{rad}}{\text{sec}^2})$$

$$+\frac{(50\ \text{kg})(9.81\frac{\text{m}}{\text{sec}^2})(4\ \text{m})}{4} = 2120\ \text{N}\cdot\text{m}$$

Summing forces in the horizontal direction

$$(\sum F_x)_{ext.} = (\sum F_x)_{eff.}$$

$$O_X = -m\frac{L}{4}\omega^2\cos\theta + m\frac{L}{4}\alpha\sin\theta$$

$$= -\frac{(50\ \text{kg})(4\ \text{m})}{4}(-5\frac{\text{rad}}{\text{s}})^2\cos 10°$$

$$+(50\ \text{kg})\frac{4\ \text{m}}{4}(14\frac{\text{rad}}{\text{s}^2})\sin 10° = -1110\ \text{N}$$

Summing forces in the vertical direction

$$(\Sigma F_y)_{ext.} = (\Sigma F_y)_{eff.}$$

$$O_y - mg = m\frac{L}{4}\omega^2 \sin\theta + m\frac{L}{4}\alpha\cos\theta$$

$$O_y = \frac{(50\,\text{kg})(4\,\text{m})}{4}(-5\frac{\text{rad}}{\text{s}})^2 \sin 10° + \frac{(50\,\text{kg})(4\,\text{m})}{4}(14\frac{\text{rad}}{\text{s}^2})\cos 10°$$
$$+ (50\,\text{kg})(9.81\frac{\text{m}}{\text{s}^2}) = 1400\,\text{N}$$

Problem 1.17 illustrates application of Newton's Laws to rigid bodies.

1.18 The disk of Figure P1.18 rolls without slipping. Assume if P = 18 N. (a) Determine the acceleration of the mass center of the disk. (b) Determine the angular acceleration of the disk.

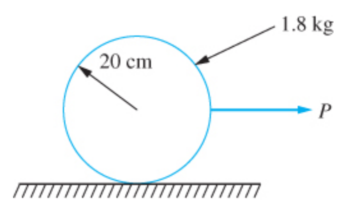


FIGURE P1.18

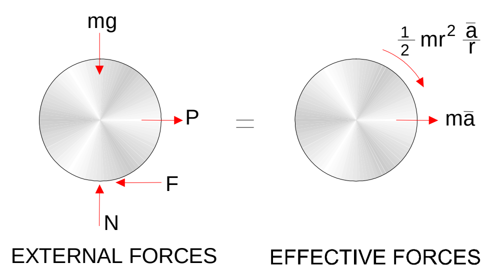
Given: m = 18 kg, P = 18 N, r = 20 cm

Find: $\bar{a}$

Solution: (a) If the disk rolls without slip then its angular acceleration is related to the acceleration of the mass center by

$$\bar{a} = r\alpha$$

Free- body diagrams of the disk at an arbitrary instant are shown below



Summing moments about the contact point

$$\left(\sum M_C\right)_{ext.} = \left(\sum M_C\right)_{eff.}$$
$$P\,r = \bar{I}\alpha + m\bar{a}r$$
$$P\,r = \frac{1}{2}mr^2\,\frac{\bar{a}}{r} + m\bar{a}r$$
$$\bar{a} = \frac{2P}{3m} = \frac{2(18\,\text{N})}{3(1.8\,\text{kg})} = 6.67\,\frac{\text{m}}{\text{s}^2}$$

(b) The angular acceleration of the disk is

$$\alpha = \frac{\bar{a}}{r} = 33.35\text{r/s}^2$$

Problem 1.18 illustrates application of Newton's Laws to a rigid body.

1.19 The coefficient of friction between the disk of Figure P1.18 and the surface is 0.12. What is the largest force that can be applied such that the disk rolls without slipping?

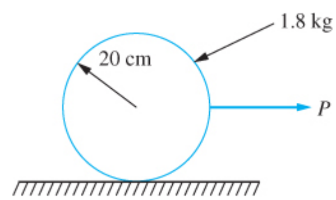
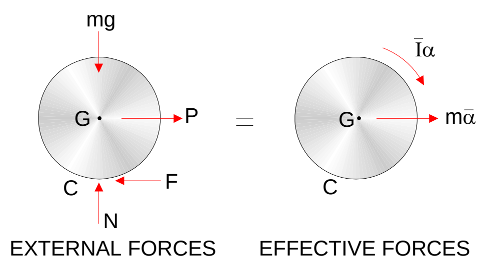


FIGURE P1.18

Given: m = 1.8 kg, r = 20 cm, μ = 0.12
Find: $P_{max.}$ for no slip

Solution: Free body diagrams of the disk at an arbitrary instant are shown below.



Summing moments about the contact point,

$$(\sum M_c)_{ext.} = (\sum M_c)_{eff.}$$
$$Pr = \bar{I}\alpha + m\bar{a}r \qquad (1)$$

If the disk rolls without slip then

$$\alpha = \frac{\bar{a}}{r} \qquad (2)$$

Substitution of eq.(2) into eq.(1) leads to

$$\bar{a} = \frac{2P}{3m} \tag{3}$$

Summing moments about the mass center of the disk

$$\left(\sum M_G\right)_{ext.} = \left(\sum M_G\right)_{eff.}$$

$$F\,r = \frac{1}{2}\,mr^2 = \frac{\bar{a}}{r}$$

$$F = \frac{1}{2}\,m\bar{a} = \frac{1}{2}\,m\,\frac{2P}{3m} = \frac{P}{3}$$

The maximum allowable friction force is μmg, thus in order for the no slip assumption to be valid,

$$\frac{P}{3} < \mu mg$$

$$P < 3\,\mu mg = 6.36\,\text{N}$$

Problem 1.19 illustrates application of Newton's Laws to a rigid body dynamics problem and rolling friction.

1.20 The coefficient of friction between the disk of Figure P1.18 and the surface is 0.12. If P = 22 N, what are the following? (a) Acceleration of the mass center. (b) Angular acceleration of the disk.

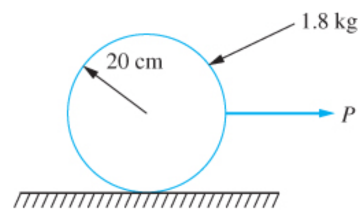
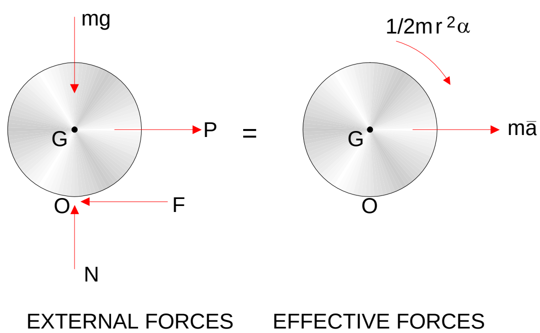


FIGURE P1.18

Given: r = 20 cm, m = 1.8 kg, P = 15 N, μ = 0.12

Find: $\bar{a}$, α

Solution: (a) Free body diagrams of the disk at an arbitrary instant of time are shown below



Summing moments about the contact point between the disk and the surface

$$\left(\sum M_O\right)_{ext.} = \left(\sum M_O\right)_{eff.}$$
$$Pr = m\bar{a}r + \frac{1}{2}mr^2\alpha \tag{1}$$

Summing moments about the mass center

$$\left(\sum M_G\right)_{ext.} = \left(\sum M_G\right)_{eff.}$$
$$Fr = \frac{1}{2}mr^2\alpha \tag{2}$$

First assume the disk rolls without slipping. Then the velocity and the acceleration of the contact point are zero, which in turn implies that $\bar{a}$ = rα. Substituting into eq.(1) yields

$$\alpha = \frac{2P}{3mr} = 27.8\,\frac{\text{rad}}{\text{s}^2}$$

If the assumption of no slip is valid, then the friction force developed is less than the maximum allowable friction force,

$$F_{\max} = \mu mg = 2.12\text{N}$$

The friction force assuming no slip is calculated using eq.(2),

$$F = \frac{1}{2}mr\alpha = \frac{1}{2}(1.8\text{ kg})(0.2\text{ m})\left(27.8\,\frac{\text{rad}}{\text{s}}\right) = 5.0\text{ N}$$

(b) Thus the disk rolls and slides. The friction force takes on its maximum permissible value of 2.12 N. The velocity of the contact point is not zero and is independent of the velocity of the mass center implying that there is no kinematic relation between the angular acceleration and the acceleration of the mass center. Setting F = 2.12 N in eq.(2) leads to

$$\alpha = \frac{2F}{mr} = \frac{2(2.12\text{ N})}{(1.8\text{ kg})(0.2\text{ m})}11.8\,\frac{\text{rad}}{\text{s}^2}$$

Problem 1.20 illustrates application of Newton's Law to a rolling rigid body. Since it is not known whether the disk slides while rolling, an assumption of no slip is made. The assumption is proved false by checking the friction force. If an assumption of rolling and slipping is first made, there is no convenient way to check the assumption.

1.21 The 3 kg block of Figure P1.21 is displaced 10 mm downward and then released from rest. (a) What is the maximum velocity attained by the 3-kg block? (b) What is the maximum angular velocity attained by the disk?

Given: $m_1 = 3$ kg, $m_2 = 5$ kg,

$I_P = 0.25$ kg-m^2, r = 20 cm,

K = 4000 N/m, $\delta = 10$ mm

Find: $\dot{x}_{max}$, ω_{max}

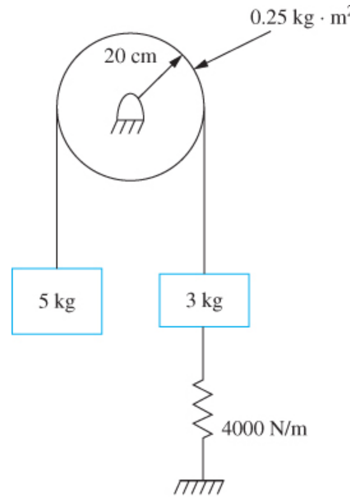


FIGURE P1.21

Solution: Since gravity and spring forces are the only external forces doing work, energy is conserved. Let position 1 refer to the position when the 3 kg block is displaced 10 mm. Let position 2 refer to the position when the velocity is a maximum. Then

$$T_1 + V_1 = T_2 + V_2 \tag{1}$$

The spring is stretched when the system is in equilibrium, due to the gravity of the blocks. Thus when the spring is in equilibrium, it has a non-zero potential energy. However, when the system is in equilibrium its total energy is zero. Thus the potential energy due to gravity balances with the potential energy due to the static deflection in the spring. Neither must be included in the analysis. With this in mind

$$T_1 = 0$$

$$V_1 = \frac{1}{2} k \delta^2 = 0.2 \mathrm{N} \cdot \mathrm{m}$$

$$V_2 = 0$$

$$T_2 = \frac{1}{2} m_1 r^2 \omega_2^2 + \frac{1}{2} m_2 r^2 \omega_2^2 + \frac{1}{2} I_P \omega_2^2 = 0.285\, \omega_2^2$$

Substitution into eq.(1) leads to

$$\omega_2 = \omega_{\max.} = 0.837 \frac{\mathrm{rad}}{\mathrm{s}}$$

Then

$$x_{max} = (0.2\ \mathrm{m})(0.837\ \mathrm{rad/s}) = 0.167\ \mathrm{m/s}$$

Problem 1.21 illustrates application of conservation of energy to a system of rigid bodies. It also illustrates that the potential energy present in a spring when a system is in equilibrium will balance with potential energies of the gravity forces that caused the static deflection. Neither must be included in a work-energy analysis as they cancel with each other.

1.22 The center of the thin disk of Figure P1.22 is displaced 15 mm and released. What is the maximum velocity attained by the disk, assuming no slipping between the disk and the surface?

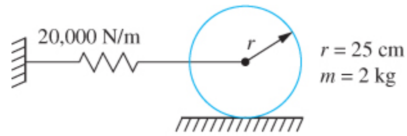


FIGURE P1.22

Given: m = 2 kg, r = 25 cm, k = 20,000 N/m, δ = 15 mm, no slip

Find: $v_{max.}$

Solution: Since the disk rolls without slipping, the velocity of the point of contact between the disk and the surface is zero, and hence the friction force does no work. Thus the spring force is the only external force which does work. The system is conservative. Let position 1 refer to the initial position of the system when the center is displaced 15 mm. Let position 2 refer to the position when the center attains its maximum velocity. Then from conservation of energy

$$T_1 + V_1 = T_2 + V_2 \tag{1}$$

where

$$T_1 = 0$$

$$V_1 = \frac{1}{2}k\delta^2 = \frac{1}{2}(20000\frac{\text{N}}{\text{m}})(0.015\text{m})^2 = 2.25\text{N}\cdot\text{m}$$

$$T_2 = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}(\frac{1}{2}mr^2)\omega_2^2$$

Since the disk rolls without slip

$$\bar{v}_2 = r\omega_2$$

and

$$T_2 = \frac{3}{4}m\bar{v}_2^2$$

Substituting into eq.(1) leads to

$$\bar{v}_2 = \bar{v}_{\text{max.}} = \sqrt{\frac{4(2.25\text{N}\cdot\text{m})}{3(2\text{ kg})}} = 1.22\frac{\text{m}}{\text{s}}$$

Problem 1.22 illustrates application of conservation of energy to a system involving a rigid body. The time history of motion for this system is examined in Chapter 4.

1.23 The block of Figure P1.23 is given a displacement δ and then released. (a) What is the minimum value of δ such that motion ensues? (b) What is the minimum value of δ such that the block returns to its equilibrium position without stopping?

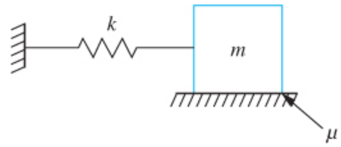


FIGURE P1.23

Given: m, k

Find: (a) value of δ for motion, (b) value of δ such that block returns to equilibrium

Solution: (a) In order for motion to occur when the block is released, the spring force must be larger than the friction force. That is

$$k\delta > \mu mg$$

$$\delta > \frac{\mu mg}{k}$$

(b) In order for the block to return to equilibrium before motion ceases, the initial potential energy stored in the spring must not be dissipated due to friction before the block returns to equilibrium. Suppose the block is given a displacement just sufficient to return it to equilibrium before motion ceases. Let position 1correspond to the initial position and position 2 correspond to the position when the block returns to its equilibrium position. The principle of work energy states

$$T_1 + V_1 + U_{1-2} = T_2 + V_2$$

Since the system is released from rest, T_1 = 0. Since the displacement is just sufficient to return the block to equilibrium, it has a zero velocity when it returns to equilibrium and T_2 = 0. Since the block is in its equilibrium position in position 2, V_2 = 0. The work done by non-conservative forces is the work done by the friction force. Thus

$$\frac{1}{2}k\delta^2 - \mu mg\delta = 0$$

$$\delta = \frac{2\mu mg}{k}$$

Problem 1.23 illustrates motion of a mass-spring system when dry fiction is present. This problem is considered again in Chapter 3 in the discussion of Coulomb damping.

1.24 The five-blade ceiling fan of Figure P1.24 operates at 60 rpm. The distance between the mass center of a blade and the axis of rotation is 0.35 m. What is the total kinetic energy?

Given: ω = 60 rpm, ceiling fan shown

Find: T

Solution: The rotational speed is converted to rad/sec by

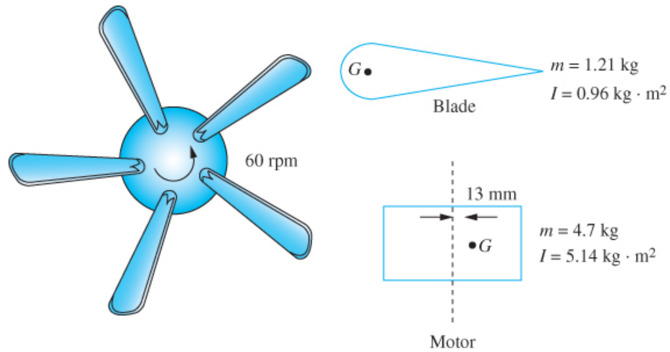


FIGURE P1.24

$$\omega = \frac{60\,\text{rev}}{1\,\text{s}}\frac{2\pi\ \text{rad}}{1\ \text{rev}}\frac{1\ \text{min}}{60\ \text{sec}} = 6.283\,\frac{\text{rad}}{\text{sec}}$$

The velocity of the mass center of the motor

$$\bar{v} = (0.013\,\text{m})\omega = 0.082\,\frac{\text{m}}{\text{s}}$$

The kinetic energy of the motor is

$$\begin{aligned} T_m &= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 \\ &= \frac{1}{2}(4.7\,\text{kg})\left(0.082\,\frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(5.14\,\text{kg}-\text{m}^2)\left(6.283\,\frac{\text{rad}}{\text{s}}\right)^2 \\ &= 101.5\ \text{N}\cdot\text{m} \end{aligned}$$

The velocity of the mass center of each blade is

$$\bar{v} = (0.35\,m)\omega = 2.20\,\frac{\text{m}}{\text{s}}$$

The kinetic energy of each blade is

$$\begin{aligned} T_b &= \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2 \\ &= \frac{1}{2}(1.21\,\text{kg})\left(2.20\,\frac{\text{m}}{\text{s}}\right)^2 + \frac{1}{2}(0.96\,\text{kg}\cdot\text{m}^2)\left(6.283\,\frac{\text{rad}}{\text{s}}\right)^2 \\ &= 21.88\ \text{N}\cdot\text{m} \end{aligned}$$

The total kinetic energy of the ceiling fan is

$$\begin{aligned} T = T_m + 5T_b &= 101.5\ \text{N}\cdot\text{m} + 5(21.88\,\text{N}\cdot\text{m}) \\ &= 210.9\,\text{N}\cdot\text{m} \end{aligned}$$

Problem 1.24 illustrates calculation of kinetic energy of a rigid body.

1.25 The U-tube manometer shown in Figure P1.25 rotates about axis *A-A* at a speed of 40 rad/sec. At the instant shown, the column of liquid moves with a speed of 20 m/sec relative to the manometer. Calculate the total kinetic energy of the column of liquid in the manometer.

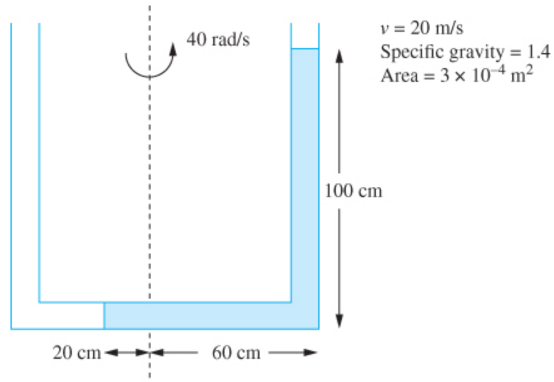
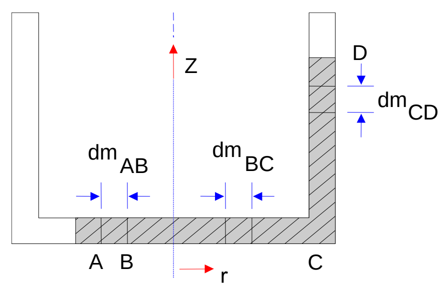


FIGURE P1.25

Given: v = 20 m/sec, ω = 40 rad/sec, A = 0.0003 m^2 , S.G.=1.4

Find: T



Solution: The column of liquid is broken into three sections. The velocity of the fluid particles comprising each section is

$$\mathbf{v}_{AB} = v\mathbf{i} + r\omega\mathbf{k}$$
$$\mathbf{v}_{BC} = v\mathbf{i} + r\omega\mathbf{k}$$
$$\mathbf{v}_{CD} = v\mathbf{j} - 0.6\,\omega\mathbf{k}$$

Consider a differential mass, defined in each part of the manometer as shown. The kinetic energy of the differential mass is

$$dT = \frac{1}{2}|\mathbf{v}|^2 dm$$

The kinetic energy of the particles in each section is obtained by integrating dT over the liquid in that section.

Section AB: $dm_{AB} = \rho A dr$

$$T_{AB} = \int_0^{0.2\,m} \frac{1}{2}\rho A\left(v^2 + \omega^2 r^2\right)dr$$
$$= \frac{1}{2}\rho A\left(0.2v^2 + 0.00267\,\omega^2\right)$$

Section BC: $dm_{BC} = \rho A dr$

$$T_{BC} = \int_0^{0.6\,m} \frac{1}{2}\rho A\left(v^2 + \omega^2 r^2\right)dr$$
$$= \frac{1}{2}\rho A\left(0.6\,v^2 + 0.072\,\omega^2\right)$$

Section CD: $dm_{CD} = \rho A dz$

$$T_{CD} = \int_0^{1\,m} \frac{1}{2}\rho A\left(v^2 + 0.36\,\omega^2\right)dz$$
$$= \frac{1}{2}\rho A\left(v^2 + 0.36\,\omega^2\right)$$

The total kinetic energy is

$$T = T_{AB} + T_{BC} + T_{CD}$$
$$= \frac{1}{2}\rho A\left(1.8\,v^2 + 0.435\,\omega^2\right)$$
$$= \frac{1}{2}(1.4)\left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(0.0003\,\text{m}^2\right)\left[1.8\left(20\,\frac{\text{m}}{\text{s}}\right)^2 + 0.435\left(40\,\frac{\text{rad}}{\text{s}}\right)^2\right]$$
$$= 297.4\ \text{N}\cdot\text{m}$$

Problem 1.25 illustrates the kinetic energy calculation of a column of liquid in a manometer. The vibrations of the column of liquid in a manometer rotating about an axis other than an axis through its center are nonlinear if the rotational speed is large enough. The differential equations are formulated using energy methods and a kinetic energy calculation similar to that developed in the solution of this problem.

1.26 The displacement function for a simply supported beam of Figure P1.26 is

$$y(x,t) = c\sin\left(\frac{\pi x}{L}\right)\cos\left(\pi^2\sqrt{\frac{EI}{\rho A L^4}}\,t\right)$$

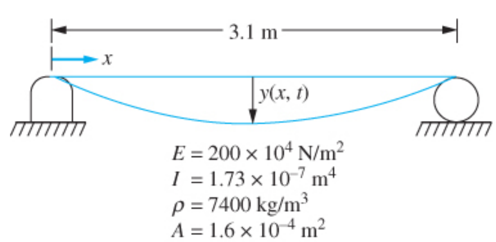


FIGURE P1.26

where $c = 0.003$ m and t is in seconds. Determine the kinetic energy of the beam.

Given: $y(x,t)$

Find: T

Solution: The kinetic energy of the beam is

$$T = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y}{\partial t}\right)^2 dx = \frac{1}{2}\int_0^L \rho A\left[c\sin\left(\frac{\pi x}{L}\right)\right]^2\left(\pi^2\sqrt{\frac{EI}{\rho A L^4}}\right)^2\left[\sin\pi^2\sqrt{\frac{EI}{\rho A L^4}}\right]^2 dx$$

$$= \frac{1}{2}c^2\pi^4\frac{EI}{L^4}\left[\sin\pi^2\sqrt{\frac{EI}{\rho A L^4}}\right]^2\int_0^L\left[\sin\left(\frac{\pi x}{L}\right)\right]^2 dx$$

$$= \frac{1}{4}c^2\pi^4\frac{EI}{L^3}\left[\sin\pi^2\sqrt{\frac{EI}{\rho A L^4}}\right]^2$$

Problem 1.26 illustrates the calculation of the kinetic energy of a continuous system.

1.27 The block of Figure P1.27 is displaced 1.5 cm from equilibrium and released.

(a) What is the maximum velocity attained by the block?
(b) What is the acceleration of the block immediately after it is released?

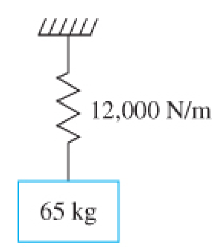


FIGURE P1.27

Given: m = 65 kg, k = 12,000 N/m, x_0 = 1.5 cm

Find: (a) v_{max} (b) a_0

Solution: When the system is in equilibrium the spring is stretched and has a static deflection Δ. Summing forces on the free-body diagram of the system's equilibrium position

$$\sum F = 0$$
$$mg - k\Delta = 0$$
$$\Delta = \frac{mg}{k} = \frac{(65\text{ kg})(9.81\text{ m/s}^2)}{12000\text{ N/m}} = 0.053\text{ m}$$

(a) Let position 0 refer to the initial position of the system. Let position 1 refer to the system when the velocity of the block is a maximum. Since the system is is released from rest in position 0, T_0=0. The total stretching in the spring in position 0 is

$$\delta_0 = x_0 + \Delta = 0.015\text{ m} + 0.053\text{ m} = 0.068\text{ m}$$

If the equilibrium plane is chosen as the datum plane for referencing the potential energy due to gravity the potential energy in position 0 is

$$V_0 = -mgx_0 + \frac{1}{2}k\delta_0^2$$
$$V_0 = -(65\text{ kg})(9.81\text{ m/s}^2) + \frac{1}{2}(12000\text{ N/m})(0.068\text{ m})^2$$
$$V_0 = 18.18\text{ N}\cdot\text{m}$$

Since all forces are conservative, application of conservation of energy is applied leading to

$$18.18\text{ N - m} = T_1 + V_1$$

The maximum kinetic energy occurs when the potential energy is a minimum, which occurs when the system passes through its equilibrium position,

$$V_1 = \frac{1}{2}k\Delta^2 = \frac{1}{2}(12000\text{ N/m})(0.053\text{ m})^2 = 16.85\text{ N}\cdot\text{m}$$

Hence

$$18.18\text{ N}\cdot\text{m} = 16.85\text{ N}\cdot\text{m} + \frac{1}{2}mv_{\max}^2$$
$$v_{\max} = 0.202\text{ m/s}$$

(b) Application of Newton’s law to the free-body diagram of the block in its initial position leads to

$$\sum F = ma_0$$
$$mg - k(x_0 + \Delta)$$
$$a_0 = g - \frac{k}{m}(x_0 + \Delta)$$
$$a_0 = 9.81\,\text{m/s}^2 - \frac{12000\ \text{N/m}}{65\ \text{kg}}(0.068\ \text{m})$$
$$a_0 = -2.74\ \text{m/s}^2$$

Problem 1.27 illustrates (a) application of conservation of energy to a particle and (b) application of Newton's law to the free-body diagram of a particle.

1.28 The slender rod of Figure P1.28 is released from the horizontal position when the spring attached at A is stretched 10 mm and the spring attached at B is unstretched. (a) What is the angular acceleration of the bar immediately after it is released? (b) What is the maximum angular velocity attained by the bar?

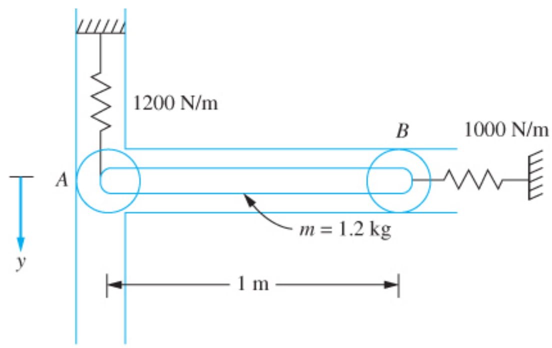
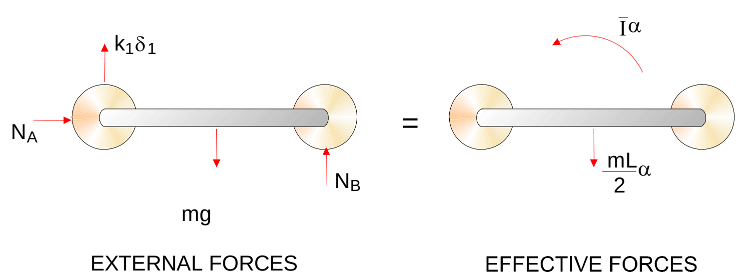


FIGURE P1.28

Given: $m = 1.2$ kg, $L = 1$m, $\delta_1 = 10$ mm, $k_1 = 1200$ N/m, $k_2 = 1000$ N/m

Find: (a) $Y_{max.}$, (b) $\omega_{max.}$

Solution: Consider first the system immediately after release.



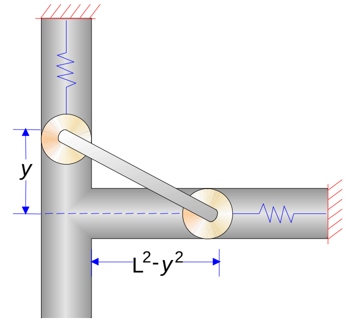
Summing moments about B

$$\left(\sum M_B\right)_{ext.} = \left(\sum M_B\right)_{eff.}$$

$$-k_1\delta_1 L + mg\frac{L}{2} = m\frac{L^2}{3}\alpha$$

$$\alpha = \frac{3}{mL}\left(\frac{mg}{2} - k_1\delta_1\right)$$

$$= -15.29\ \frac{\text{rad}}{\text{s}^2}$$



Hence α is clockwise and the bar moves upward. Now consider the geometry of the bar when it has moved a distance y upward. The horizontal displacement of B is

$$\delta_B = L - \sqrt{L^2 - y^2}$$

(b) Let ω be the angular velocity of the bar. Then using the relative velocity equation

$$\mathbf{v}_A = v_A\mathbf{j} = v_B\mathrm{i} + \omega\mathbf{k}\mathbf{x}\left(-L\cos\theta\mathbf{i} - L\sin\theta\mathbf{j}\right)$$
$$v_A\mathbf{j} = \left(v_B + L\omega\sin\theta\right)\mathbf{i} - L\omega\cos\theta\mathbf{j}$$

From the x component of the above equation

$$v_B = -L\sin\theta\,\omega$$

The velocity of the mass center of the bar is

$$\mathbf{v} = -L\omega\sin\theta\mathbf{i} + \omega\mathbf{k}\mathbf{x}\left(-\frac{L}{2}\cos\theta\mathbf{i} - \frac{L}{2}\sin\boldsymbol{\theta}\mathbf{j}\right)$$

$$\vec{v} = -\frac{L}{2}\omega\sin\theta\mathbf{i} + -\frac{L}{2}\omega\cos\theta\mathbf{j}$$

$$|\mathbf{v}| = \frac{L}{2}\omega$$

Let position 2 refer to the position of the system when the angular velocity is a maximum. Energy is conserved between position 1 and position 2.

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}k_1\delta_1^2 = mg\frac{y}{2} + \frac{1}{2}k_1(\delta_1 - y)^2 + \frac{1}{2}k_2\left(L - \sqrt{L^2 - y^2}\right) + \frac{1}{2}\frac{1}{12}mL^2\omega^2 + \frac{1}{2}m\left(\frac{L}{2}\omega\right)^2 \quad (3)$$

$$0 = \left(m\frac{g}{2} + k_1\delta_1\right)y + \frac{1}{2}k_1y^2 + \frac{1}{2}k_2\left(2L^2 - y^2 + 2L\sqrt{L^2 - y^2}\right) + \frac{1}{6}mL^2\omega^2$$

The above equation could be expressed as

$$T_2 = \frac{1}{6} mL^2 \omega^2 = V_1 - V_2$$

Thus the maximum angular velocity occurs when V_1-V_2 is maximized. To this end

$$\frac{d}{dy}(V_1 - V_2) = 0 = k_1\delta_1 - \frac{mg}{2} + (k_2 - k_1)y - \frac{k_2 Ly}{\sqrt{L^2 - y^2}}$$

$$6.114 - 200y - \frac{1000y}{\sqrt{1 - y^2}} = 0$$

A trial and error solution of the above equation reveals that the maximum angular velocity occurs for y = 0.0051 m. Then from eq. (3),

$$\omega = 0.322 \frac{\text{rad}}{\text{s}}$$

Problem 1.28 illustrates application of conservation of energy to a rigid body system.

1.29 Let x be the displacement of the left end of the bar of the system in Figure P1.29. Let θ represent the clockwise angular rotation of the bar. (a) Express the kinetic energy of the system at an arbitrary instant in terms of $\dot{x}$ and $\dot{\theta}$. (b) Express the potential energy of an arbitrary instant in terms of x and θ.

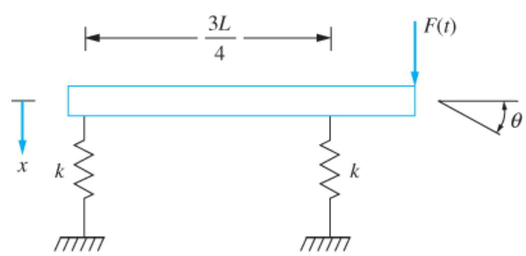


FIGURE P1.29

Given: x and θ as generalized coordinates

Find: (a) T (b) V

Solution: (a) The kinetic energy of a rigid body is

$$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$$

The angular velocity of the bar is $\omega = \dot{\theta}$. The displacement of the mass center in terms of the chosen generalized coordinates is

$$\bar{x} = x + \frac{L}{2}\sin\theta$$

Thus the velocity of the mass center is

$$\dot{\bar{x}} = \dot{x} + \frac{L}{2}\dot{\theta}\cos\theta$$

Hence the kinetic energy of the system at an arbitrary instant is

$$T = \frac{1}{2}m\left(\dot{x} + \frac{L}{2}\dot{\theta}\cos\theta\right)^2 + \frac{1}{2}\bar{I}\dot{\theta}^2$$

If the small-angle assumption is used the kinetic energy of the linearized system is

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}mL\dot{x}\dot{\theta} + \frac{1}{2}\left(m\frac{L^2}{4} + \bar{I}\right)\dot{\theta}^2$$

(b) The potential energy is due to the springs and is

$$V = \frac{1}{2}kx^2 + \frac{1}{2}k\left(x + \frac{3L}{4}\theta^2\right)$$

Problem 1.29 illustrates the evaluation of the kinetic energy and potential energy of a rigid body at an arbitrary instant in terms of chosen generalized coordinates.

1.30 Repeat problem 1.29 using coordinates x_1, which is the displacement of the mass center, and x_2, which is the displacement of the point of attachment of the spring that is a distance 3*L*/4 from the left end.

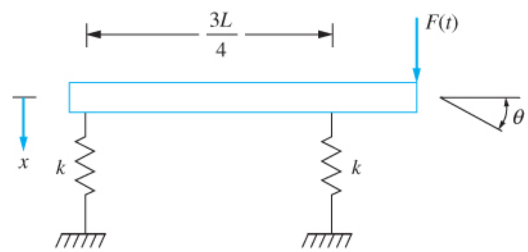


FIGURE P1.29

Given: x_1 and x_2 as generalized coordinates.

Find: (a) T (b) V

Solution: The kinetic energy of the bar at an arbitrary instant is

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}I\left[\left(\frac{4}{L}\right)(\dot{x}_2 - \dot{x}_1)\right]^2$$

The potential energy of the bar at an arbitrary instant is

$$V = \frac{1}{2}k(3x_2 - 2x_1)^2 + \frac{1}{2}k{x_2}^2$$

Problem 1.30 illustrates the evaluation of the kinetic and potential energy of a rigid body at an arbitrary instant in terms of chosen generalized coordinates.

1.31 Let θ represent the clockwise angular displacement of the pulley system in Figure P1.31 from the system's equilibrium position.

(a) Express the potential energy of the system at an arbitrary instant in terms of θ.

(b) Express the kinetic energy of the system at an arbitrary instant in terms of $\dot{\theta}$.

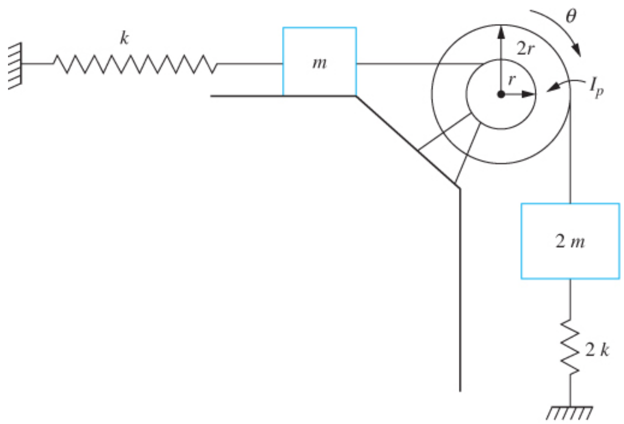


FIGURE P1.31

Given: θ as generalized coordinate

Find: (a) V (b) T

Solution: Consider the free-body diagram of the system in its equilibrium position. Summing moments about the center of the pulley

$$\sum M_C = 0$$
$$-k\Delta_1 r - 2k\Delta_2(2r) + 2mg(2r) = 0$$

From the geometry of the system

$$\theta_{st} = \frac{\Delta_1}{r} = \frac{\Delta_2}{2r}$$

which when substituted into the previous equation leads to

$$\Delta_1 = \frac{4mg}{9k} \qquad \Delta_2 = \frac{8mg}{9k}$$

Let x_1 represent the displacement of the sliding block from the system's equilibrium position. Let x_2 represent the displacement of the hanging block from the system's equilibrium position. From geometry

$$x_1 = r\theta \qquad x_2 = 2r\theta$$

(a) Choosing the equilibrium position of the system as the datum for potential energy calculations, the potential energy at an arbitrary instant is

$$V = \frac{1}{2}k(x_1 + \Delta)^2 + \frac{1}{2}2k(x_2 + \Delta_2)^2 - 2mgx_2$$
$$V = \frac{1}{2}k\left(r\theta + \frac{4mg}{9k}\right)^2 + \frac{1}{2}2k\left(2r\theta + \frac{8mg}{9k}\right)^2 - 2mgx_2$$

Simplification leads to

$$V=\frac{9}{2}kr^2\theta^2+\frac{8}{9}\left(\frac{mg}{k}\right)^2$$

(b) The kinetic energy of the system at an arbitrary instant is

$$T=\frac{1}{2}m\dot{x}_1^2+\frac{1}{2}I_p\dot{\theta}^2+\frac{1}{2}2m\dot{x}_2^2$$

$$T=\frac{1}{2}m(r\dot{\theta})^2+\frac{1}{2}I_p\dot{\theta}^2+\frac{1}{2}2m(2r\dot{\theta})^2$$

$$T=\frac{1}{2}(9mr^2+I_p)\dot{\theta}^2$$

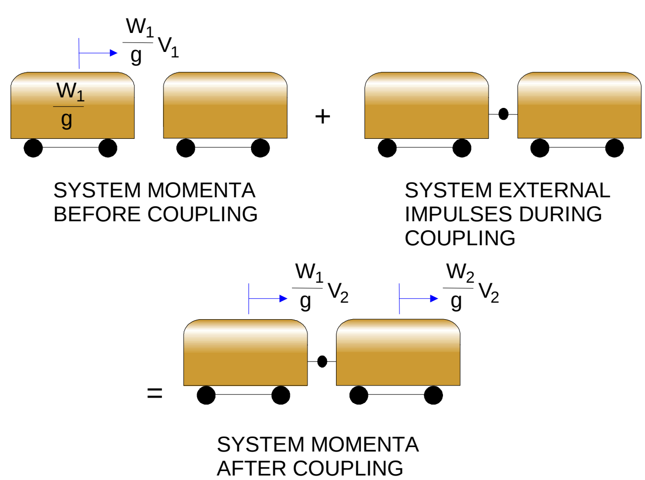
Problem 1.31 illustrates the calculation of a potential and kinetic energy of a system of rigid bodies at an arbitrary instant in terms of a chosen generalized coordinate.

1.32 A 20 ton railroad car is coupled to a 15 ton car by moving the 20 ton car at 5 mph toward the stationary 15 ton car. (a) What is the resulting speed of the two-car coupling? (b) What would the resulting speed be if the 15 ton car is moving at 5 mph toward a stationary 20 ton car?

Given: $W_1 = 40000$ lb, $W_2 = 30000$ lb, $v_1 = 5$ mph

Find: v_2

Solution: (a) Consider the impulse and momentum diagrams below



There are no external impulses acting on the two car system during coupling. Applying the principle of linear impulse and linear momentum

$$\frac{W_1}{g}v_1 = \left(\frac{W_1}{g} + \frac{W_2}{g}\right)v_2$$

$$v_2 = \frac{W_1 v_1}{W_1 + W_2}$$

$$= \frac{(40000\,\text{lb})(5\,\text{mph})}{70000\,\text{lb}}$$

$$= 2.86\,\text{mph}$$

(b) If the 15 ton car has a velocity of 5 mph the velocity of the system after coupling is

$$v_2 = \frac{(30000\text{ lb})(5\text{ mph})}{(70000\text{ lb})} = 2.14\text{ mph}$$

Problem 1.32 illustrates application of the principle of linear impulse and linear momentum to a system when linear momentum is conserved. The couplings between railroad cars are actually elastic. Thus, after coupling the cars move relative to one another. The two car system will move together with a rigid body motion, but relative motion will occur. This is an example of a unrestrained system considered in Chapters 6 and 7.

1.33 The 15 kg block of Figure P1.33 is moving with a velocity of 3 m/s at t = 0 when the force $F(t)$ is applied to the block. (a) Determine the velocity of the block at t = 2 s. (b) Determine the velocity of the block at t = 4 s. (c) Determine the block's kinetic energy at t = 4 sec.

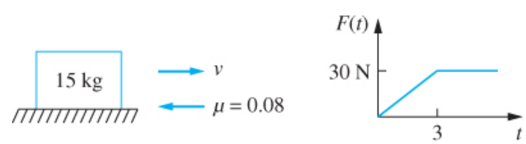


FIGURE P1.33

Given: $m = 15$ kg, $v_1 = 3$ m/s, $\mu = 0.08$, F(t)

Find: (a) v(t=2 s) (b) v(t=4 s) (c) T

Solution: (a) The principle of impulse and momentum is used to determine the velocity at t=2 s. Application of the principle leads to

$$mv_1 + \int_0^2 [F(t) - \mu mg]dt = mv_2$$

or substituting in given numbers yields

$$(15\text{ kg})\left(3\frac{\text{m}}{\text{s}}\right) + \frac{1}{2}(20\text{ N})(2\text{ s}) - (0.08)(15\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\text{ s}) = (15\text{ kg})v_2$$

$$v_2 = 2.76\text{ m/s}$$

(b) The velocity at t = 4 s is determined from the principle of impulse and momentum

$$mv_1 + \int_0^4 [F(t) - \mu mg]dt = mv_2$$

which upon substitution of given numbers yields

$$(15\text{ kg})\left(3\frac{\text{m}}{\text{s}}\right) + \frac{1}{2}(30\text{ N})(3\text{ s}) + (30\text{ N})(1\text{ s}) - (0.08)(15\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(4\text{ s}) = (15\text{ kg})v_2$$

$$v_2 = 4.86\text{ m/s}$$

(c) The kinetic energy is

$$T = \frac{1}{2}mv_2^2 = \frac{1}{2}(15\text{ kg})(4.86\text{ m/s})^2 = 177.1\text{ J}$$

Problem 1.33 illustrates application of the principle of impulse and momentum.

1.34 A 400 kg forging hammer is mounted on four identical springs, each of stiffness k = 4200 N/m. During the forging process, a 110 kg hammer, which is part of the machine, is dropped from a height of 1.4 m onto an anvil, as shown in Figure P1.34. (a) What is the resulting velocity of the entire machine after the hammer is dropped? (b) What is the maximum displacement of the machine?

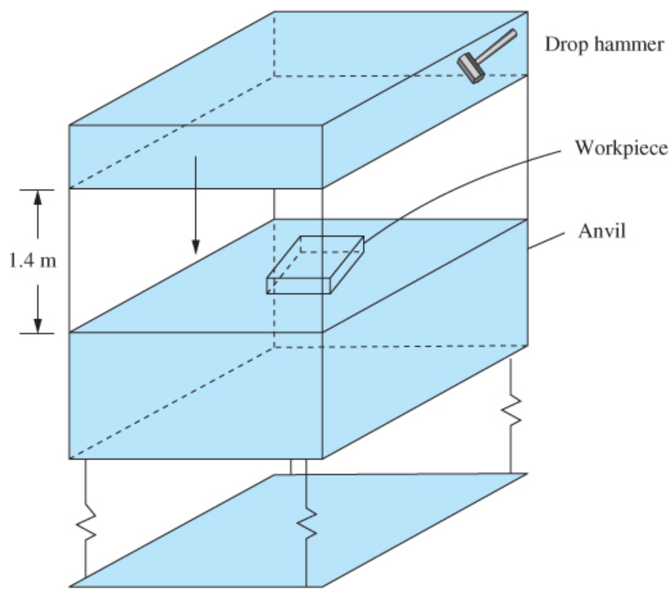


FIGURE P1.34

Given: m = 400 kg, k = 42000 N/m, m_h = 110 kg, h = 1.4 m

Find: (a) v (b) x_{max}

Solution: (a) Application of the principle of conservation of energy to the hammer as it drops leads to the velocity of the hammer immediately before impacting the anvil

$$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.4\text{ m})} = 5.24\ \frac{\text{m}}{\text{s}}$$

Applying the principle of impulse and momentum to the hammer and anvil as the hammer strikes leads to (assuming the hammer is part of the machine and the hammer sticks to and moves with the machine)

$$m_h v = mv_2 \Rightarrow v_2 = \frac{m_h v}{m} = \frac{(110\text{ kg})(5.24\text{ m/s})}{400\text{ kg}} = 1.44\text{ m/s}$$

(b) Application of the principle of conservation of energy between the time immediately after impact to the time when the machine reaches its maximum displacement

$$\frac{1}{2}mv_2^2 = \frac{1}{2}kx_{max}^2$$

$$x_{max} = \frac{m}{k}v_2 = \frac{400\ \text{kg}}{4200\ \text{N/m}}1.44\frac{\text{m}}{\text{s}} = 0.137\ \text{m}$$

Problem 1.34 illustrates application of the principle of impulse and momentum and the principle of conservation of energy.

1.35 The motion of a baseball bat in a ballplayer's hands is approximated as a rigid-body motion about an axis through the player's hands, as shown in Figure P1.35. The bat has a centroidal moment of inertia *I*. The player's "bat speed" is ω, and the velocity of the pitched ball is *v*. Determine the distance from the player's hand along the bat where the batter should strike the ball to minimize the impulse felt by his/her hands. Does the distance change if the player "chokes up" on the bat, reducing the distance from *G* to his/her hands?

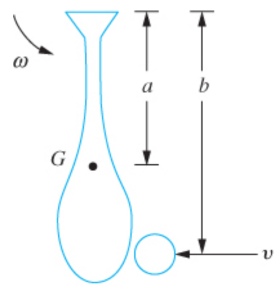
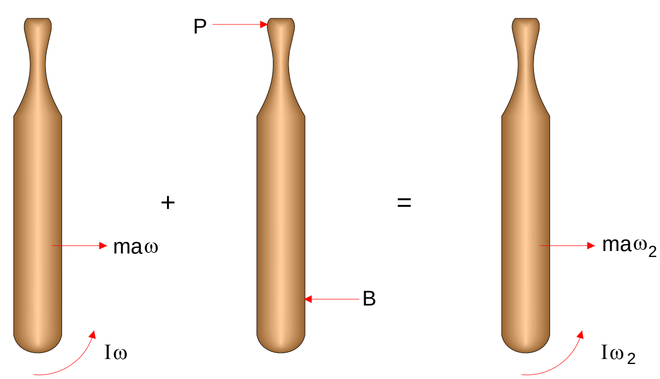


FIGURE P1.35

Given: I, a, v, ω, m

Find: b

Solution: When the bat strikes the pitched ball, the ball exerts an impulse on the bat, call it B. Since the batter is holding the bat, he feels an impulse, call it P. The effect of the impulse on the bat is to change the "bat speed" from ω before hitting the ball to ω_2 after hitting the ball. Impulse-momentum diagrams of the bat during this time are shown below.



SYSTEM MOMENTA BEFORE STRIKING BALL + SYSTEM EXTERNAL IMPULSES DURING STRIKING = SYSTEM MOMENTA AFTER STRIKING BALL

Applying the principle of linear impulse and linear momentum

$$
\begin{aligned}
ma\omega + P - B &= ma\omega_2 \\
B &= ma(\omega - \omega_2) + P
\end{aligned}
\qquad (1)
$$

Applying the principle of angular impulse and angular momentum about an axis through the batter's hands gives

$$
\begin{aligned}
\mathrm{I}\omega + ma^2\omega - Bb &= \mathrm{I}\omega_2 + ma^2\omega_2 \\
B &= \frac{1}{b}\left(\mathrm{I} + ma^2\right)\left(\omega - \omega_2\right)
\end{aligned}
\qquad (2)
$$

Equating B from eqs. (l) and (2) leads to

$$P = (\omega - \omega_2)\left(\frac{\mathrm{I} + ma^2}{b} - ma\right)$$

Note that P = 0 if

$$b = a + \frac{\mathrm{I}}{ma}$$

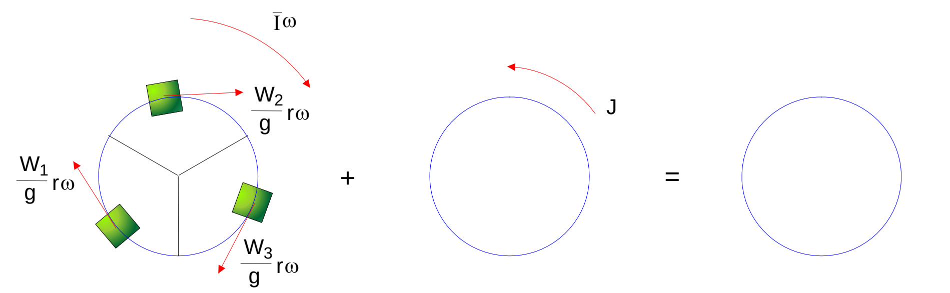
Problem 1.35 illustrates application of the principle of linear impulse and momentum and angular impulse and momentum. The location where the bat should strike the ball to minimize the impulse felt by the batter is called the center of percussion.

1.36 A playground ride has a centroidal moment of inertia of 17 slug · ft^2. Three children of weights 50 lb, 50 lb, and 55 lb are on the ride, which is rotating at 60 rpm. The children are 30 in. from the center of the ride. A father stops the ride by grabbing it with his hands. What angular impulse is felt by the father?

Given: I = 17 slugs-ft^2, W_1 =50 lb, W_2 = 50 lb, W_3 = 55 lb, r = 20 in, ω = 60 rpm = 6.48 rad/sec

Find: J to stop the ride.

Solution: The father applies an angular impulse about the center of the ride of magnitude J to stop the ride. Consider the impulse and momentum diagrams



SYSTEM MOMENTA BEFORE FATHER STOPS RIDE

EXTERNAL IMPULSE APPLIED BY FATHER

SYSTEM MOMENTA AFTER FATHER STOPS RIDE

The principle of angular impulse and angular momentum about the center of the ride is

$$\begin{pmatrix}\text{angular momentum}\\ \text{about O before}\\ \text{impulse}\end{pmatrix} + \begin{pmatrix}\text{applied angular}\\ \text{impulse about O}\end{pmatrix} = \begin{pmatrix}\text{angular mometum}\\ \text{about O after}\\ \text{impulse}\end{pmatrix}$$

$$\bar{I}\omega + \frac{W_1}{g} r\omega(r) + \frac{W_2}{g} r\omega(r) + \frac{W_3}{g} r\omega(r) - J = 0$$

$$J = \left[\bar{I} + \frac{1}{g}(W_1 + W_2 + W_3)r^2\right]\omega$$

$$J = \left[17\,\text{sulgs}\cdot\text{ft}^2 + \frac{155\,\text{lb}}{32.2\,\frac{\text{ft}}{\text{s}^2}}(1.667\,\text{ft})^2\right]\left(6.48\,\frac{\text{rad}}{\text{s}}\right)$$
$$= 197.1\,\text{N}\cdot\text{sec}\cdot\text{m}$$

Problem 1.36 illustrates application of the principle of angular impulse and angular momentum.

1.37 The natural frequencies of a thermally loaded fixed-fixed beam (Figure P1.37) are a function of the material properties of the beam, including:

E, the elastic modulus of the beam

ρ, the mass density of the beam

α, the coefficient of thermal expansion

The geometric properties of the beam are

A, its cross-sectional area

I, its cross section moment of inertia

L, its length

Also,

ΔT, the temperature difference between the installation and loading

(a) What are the dimensions involved in each of the parameters?
(b) How many dimensionless parameters does the Buckingham Pi theorem predict are in the non-dimensional formulation of the relation between the natural frequencies and the other parameters?
(c) Develop a set of dimensionless parameters.

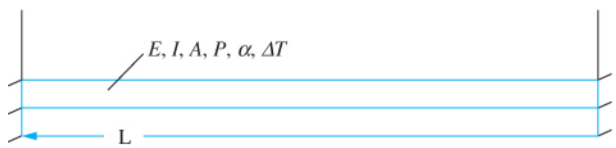


FIGURE P1.37

Solution: (a) The dimensions of the parameters are

$$\text{E: F/L}^2 \quad \rho\text{:}\, \frac{\text{M}}{\text{L}^3} = \frac{\text{FT}^2}{\text{L}^4} \quad \alpha\text{:}\, \frac{L}{\Theta} \quad \text{A:L}^2 \quad \text{I: L}^4 \quad \text{L: L} \quad \Delta T\text{:}\, \Theta \quad \omega_n = \frac{1}{\text{T}}$$

where M represents mass, L represents length, T represents time, and Θ represents temperature.

(b) The Buckingham Pi theorem implies that there are n=m-k dimensionless parameters in the formulation where m is the number of dimensional parameters and k is the number of basic dimensions in those variables. There are 8 dimensional parameters and 4 basic dimensions in the parameters which implies there are 4 nondimensional parameters.

(c) Dimensionless parameters are $\Pi_1 = \frac{\varrho A \omega_n^2}{E}, \Pi_2 = \frac{\alpha \Delta T}{L}, \Pi_3 = \frac{AL^2}{I}, \Pi_4 = \frac{A}{L^2}$

Problem 1.37 illustrates application of the Buckingham Pi theorem.

1.38 The drag force *F* on a circular cylinder due to vortex shedding is a function of

U, the velocity of the flow

μ, the dynamic viscosity of the fluid

ρ, the mass density of the fluid

L, the length of the cylinder

D, the diameter of the cylinder

(a) What are the dimensions involved in each of the parameters?
(b) How many dimensionless parameters does the Buckingham Pi theorem predict are in the non-dimensional formulation of the relation between the natural frequencies and the other parameters?
(c) Develop a set of dimensionless parameters.

Solution: (a) Dimensions of the parameters are

$$\mathrm{U}:\frac{L}{T} \quad \mu:\frac{FT}{L^2} \quad \rho:\frac{\mathrm{M}}{\mathrm{L}^3}=\frac{\mathrm{FT}^2}{\mathrm{L}^4} \quad L:\mathrm{L} \;\; D:\mathrm{L} \;\; F:\mathrm{F}$$

(b) The Buckingham Pi theorem implies that there are n=m-k dimensionless parameters in the formulation where m is the number of dimensional parameters and k is the number of basic dimensions in those variables. There are 6 dimensional parameters and 3 basic dimensions in the parameters which implies there are 3 nondimensional parameters.

(c) Dimensionless parameters are $\Pi_1 = \frac{\varrho AL}{\mu}$, $\Pi_2 = \frac{D}{L}$, $\Pi_3 = \frac{F}{\rho U^2 A}$

Problem 1.38 illustrates use of the Buckingham Pi Theorem.

1.39 The principal normal stress σ due to forcing of a beam with a concentrated harmonic excitation is a function of

F_0, the amplitude of loading

ω, the frequency of the loading

E, the elastic modulus of the beam

ρ, the mass density of the beam

A, the beam's cross-sectional area

I, the beam's cross-sectional moment of inertia

L, the beam's length

a, the location of the load along the axis of the beam

(a) What are the dimensions involved in each of the parameters?
(b) How many dimensionless parameters does the Buckingham Pi theorem predict are in the non-dimensional formulation of the relation between the natural frequencies and the other parameters?
(c) Develop a set of dimensionless parameters.

Solution: (a) Dimensions of the parameters are

$$F_0\text{:F} \quad \omega\text{: } \frac{1}{T} \quad \rho\text{: } \frac{\mathrm{M}}{\mathrm{L}^3} = \frac{\mathrm{FT}^2}{\mathrm{L}^4} \ \text{E:} \frac{\mathrm{F}}{\mathrm{L}^2} \quad L\text{: L} \ \ A\text{: L}^2 \ \ \text{I:}L^2 \ \alpha\text{:L} \ \ \sigma\text{:} \frac{\mathrm{F}}{\mathrm{L}^2}$$

(b) The Buckingham Pi theorem implies that there are n=m-k dimensionless parameters in the formulation where m is the number of dimensional parameters and k is the number of basic dimensions in those variables. There are 9 dimensional parameters and 3 basic dimensions in the parameters which implies there are 6 nondimensional parameters.

(c) Dimensionless parameters are

$$\Pi_1 = \frac{\sigma}{E}, \Pi_2 = \frac{\alpha}{L}, \Pi_3 = \frac{F_0}{\rho I \omega^2}, \Pi_4 = \frac{A}{L^2}, \Pi_3 = \frac{I}{AL^2}, \Pi_6 = \frac{F_0}{EL^2}$$

Problem 1.39 illustrates use of the Buckingham Pi theorem.

1.40 A MEMS system is undergoing simple harmonic motion according to

$$x(t) = [3.1\sin(2 \times 10^5 t + 0.48) + 4.8\cos(2 \times 10^5 t + 1.74)]\mu\text{m}$$

(a) What is the period of motion? (b) What is the frequency of motion in Hz? (c) What is the amplitude of motion? (d) What is the phase and does it lead or lag? (e) Plot the displacement.

Given: $x(t)$

Find: (a) T (b) f (c) A (d) ϕ

Solution: (a) The period is $T = \frac{2\pi}{2\times10^5} = 31.4\ \mu s$

(b) The frequency is the reciprocal of the period, $f = \frac{1}{31.4\ \mu s} = 3.183 \times 10^4$ Hz.

(c) The amplitude is obtained by writing the response in the form of $x(t) = A\sin(2 \times 105 t + \phi$. To this end

$$\begin{aligned} &3.1\sin(2 \times 10^5 t + 0.48) + 4.8\cos(2 \times 10^5 t + 1.74) \\ &\qquad = 3.1[\sin(2 \times 10^5 \mathrm{t})\cos 0.48 + \cos(2 \times 10^5 \mathrm{t})\sin 0.48] \\ &\qquad + 4.8[\cos(2 \times 10^5 \mathrm{t})\cos 1.74 - \sin(2 \times 10^5 \mathrm{t})\sin 1.74] \end{aligned}$$

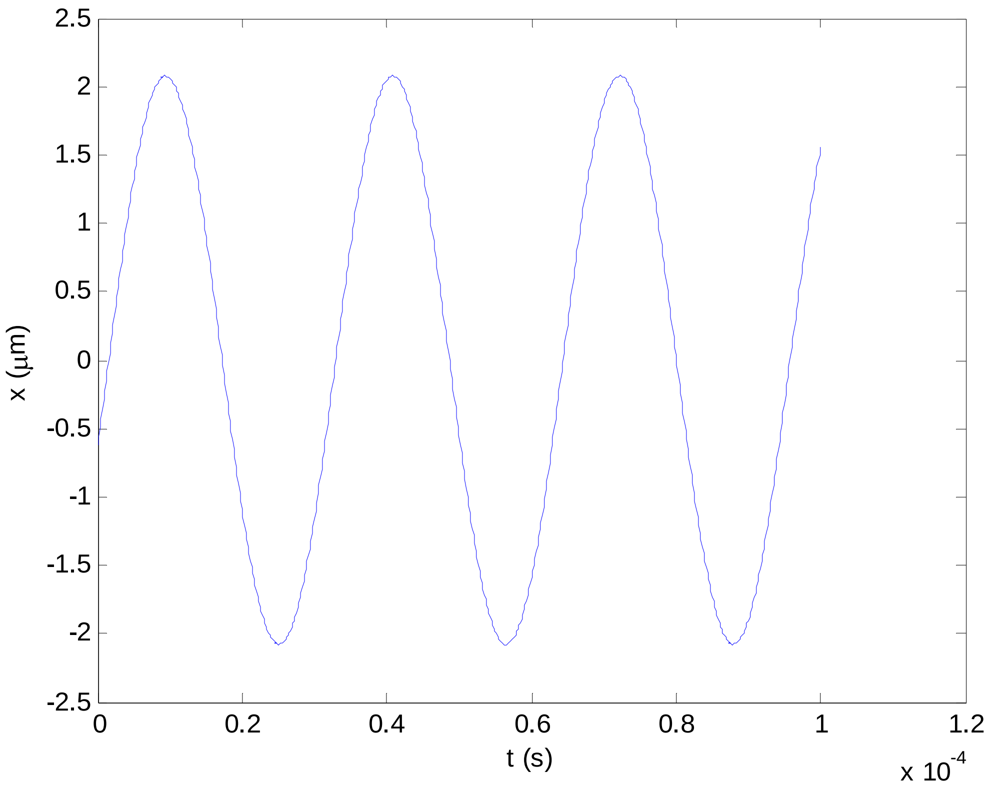
$$= (3.1\cos 0.48 - 4.8\sin 1.74)\sin(2 \times 10^5 \mathrm{t}) + (3.1\sin 0.48 + 4.8\cos 1.74)\cos(2 \times 10^5 \mathrm{t})$$

$$= -1.918\sin(2 \times 10^5 \mathrm{t}) + 0.6232\cos(2 \times 10^5 \mathrm{t})$$

$$= 2.0774\sin(2 \times 10^5 \mathrm{t} - 0.3047)$$

(d) The amplitude is 2.0774 μm. The phase is -0.3047 rad and is a phase lag.

(e)



Problem 1.40 illustrates simple harmonic motion.

1.41 The force that causes simple harmonic motion in the mass-spring system of Figure P1.31 is $F(t) = 35\sin 100t$ N. The resulting displacement of the mass is $x(t) = 0.002\sin(30t - \pi)$ m. (a) What is the period of the motion? (b) The amplitude of displacement is $X = \frac{F_0}{k}M$ where F_0 is the amplitude of the force and M is a dimensionless factor called the magnification factor. Calculate M. (c) M has the form

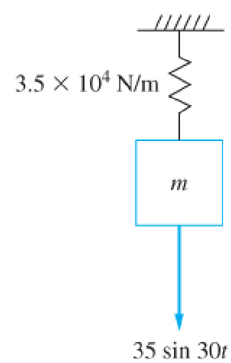


FIGURE P1.41

$$M = \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$$

where ω_n is called the natural frequency. If $\omega_n < \omega$, then $\phi = \pi$; otherwise $\phi = 0$. Calculate ω_n.

Given: $F(t), x(t), k = 3.5 \times 10^4$ N/m

Find: (a) T , (b) M , (c) ω_n

Solution: (a) The period of motion is $T = \frac{2\pi}{30} = 0.2094$ s.

(b) $M = \frac{kX}{F_0} = \frac{\left(3.5\times10^4 \frac{\text{N}}{\text{m}}\right)(0.002 \text{ m})}{35 \text{ N}} = 2$

(c) Since $\phi = \pi$, $\omega_n < \omega$ and

$$M = \frac{1}{\left(\frac{\omega}{\omega_n}\right)^2 - 1} \Longrightarrow \left(\frac{\omega}{\omega_n}\right)^2 = 1 + \frac{1}{M} = 1.5 \Longrightarrow \omega_n = \frac{\omega}{\sqrt{1.5}} = \frac{30 \ \frac{\text{rad}}{\text{s}}}{\sqrt{1.5}} = 24.49 \ \frac{\text{rad}}{\text{s}}$$

Problem 1.41 illustrates simple harmonic motion.

1.42 The displacement vector of a particle is

$$\mathbf{r}(t) = [2\sin 20t \ \ \mathbf{i} + 3\cos 20t \ \ \mathbf{j}\,] \text{ mm}$$

(a) Describe the trajectory of the particle. (b) How long does it take the particle to make one circuit around the path?

Given: $\mathbf{r}(t)$

Find: path of particle, t

Solution: From the given information $x(t) = 2\sin 20t$ and $y(t) = 3\cos 20t$. Eliminating t between the equations leads to

$$x^2 + \frac{4}{9}y^2 = 1$$

The time it takes to make one circuit around the elliptical path is

$$t = \frac{2\pi}{20} = 0.314 \text{ s}$$

Problem 1.42 illustrates the trajectory of a particle undergoing simple harmonic motion in x and y.